Uluğ Çapar (FENS, Sabanci University, Tuzla, Istanbul,Turkey)

On the Solution of a Non-linear Stochastic Prey-Predator Equation

This paper is basically the stochastic version of [1]. The related deterministic prey-predator model is the following semi-linear hyperbolic system in two variables

$$D_1 u_1(x,t) = (\partial_t + c_1\partial_x)u_1(x,t) = \lambda_1 u_1(x,t)u_2(x,t)$$

$$D_2 u_2(x,t) = (\partial_t + c_2\partial_x)u_2(x,t) = \lambda_2 u_1(x,t)u_2(x,t)$$

$$a_j(x) = u_j(x,0) = \Delta_j\delta(x-\xi_j)\ ; \qquad j=1,2, \quad \lambda_1\lambda_2 < 0$$

As the initial data are measures, u_1 and u_2 are expected to be as singular as the initial data, thus the product necessitates a non-linear theory of distributions like that of Colombeau. For the stochastic case driven by the white noise a more realistic model could be

$$D_1 u_1 = \lambda_1(u_1 + \dot{B}_1)(u_2 + \dot{B}_2)$$

$$D_2 u_2 = \lambda_2(u_1 + \dot{B}_1)(u_2 + \dot{B}_2)$$

$\dot{B}_1, \dot{B}_2 \in (S)^*$ (Hida distributions), their product is not defined in $(S)^*$ but meaningful in the stochastic Colombeau distribution $\mathcal{G}_s$ along the lines introduced in [2] and [3]. We try to solve the last sytem at the representatives platform where $u_1 = \alpha + \mathcal{N}_s, u_2 = \beta + \mathcal{N}_s$ then show that $\alpha, \beta \in \mathcal{E}_{M,s}$. Here $\mathcal{N}_s$ and $\mathcal{E}_{M,s}$ are the ideal of null germs and the algebra of moderate members of the Colombeau algebra respectively. Hence their classes will form a generalized solution. Considering

$$D_1 u_1 = \lambda_1(u_1 + \alpha)(u_2 + \beta)$$

$$D_2 u_2 = \lambda_2(u_1 + \alpha)(u_2 + \beta)$$

and noticing that $D_1(\lambda_2 u_1) = D_2(\lambda_1 u_2)$, there exists X with $D_2 X = \lambda_2 u_1$, $D_1 X = \lambda_1 u_2$. Following the Hashimoto's method let $Y = e^{-X}$ and that $\lambda_1 = 1 = -\lambda_2$ for simplicity, we find that Y satisfies the wave equation

$$Y_{tt} - Y_{xx} + (\alpha+\beta)Y_x + (\alpha-\beta)Y_t - \alpha\beta Y = 0.$$

With the transformation $\xi = x - t, \eta = x + t$:

$$Y_{\xi\eta} - \frac{1}{2}(\beta Y_\xi + \alpha Y_\eta) + \frac{1}{4}\alpha\beta Y = 0.$$

Separation of variables $Y_{\xi\eta} = F(\xi)G(\eta)$ yields

$$(F'(\xi) - \frac{1}{2}\alpha F(\xi))(G'(\eta) - \frac{1}{2}\beta G(\eta)) = 0.$$

Under the natural and reasonable assumption that white noises propagate along the two families of characteristics, i.e. $\alpha = \alpha(x-t), \beta = \beta(x+t)$, the system is completeley solvable.

[1] Colombeau,J.F.; Oberguggenberger, M.: On a hyperbolic system with a compatible quadratic term : Generalized solutions, delta waves and multiplication of distributions, J.Commun. Partial Differ. Equations **15**, no.7, 905-938 (1990).

[2] U.Çapar; H. Aktuğlu : A new construction of random Colombeau distributions, Statistics and Probability Letters **54** ,291-299 (2001).

[3] U.Çapar ; H. Aktuğlu : An overall view of stochastics in Colombeau related algebras , Stochastic Analysis and Related Topics viii, Progress in Probability, vol.**53**, U.Çapar, A.S.Ustunel Editors, 67-90, Birkhauser (2003).