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About derivations in algebras of measurable operators

Let M be an von Neumann algebra, let $LS(M)$ be the $*$-algebra of all locally measurable operators affiliated with to M and let $S(M)$ be the $*$-algebra of all measurable operators affiliated with to M. Let τ be a semi-finite normal faithful trace on M and $S(M,\tau)$ be the $*$-algebra of all τ-measurable operators affiliated with to M ([1]).

Let A be an algebra. The linear operator δ on A is called *derivation* if $\delta(ab) = \delta(a)b + a\delta(b)$ for any $a, b \in A$. The derivation δ is called *inner* if $\delta(x) = [d, x] = dx - xd$ when $d \in A$.

Theorem 1. Let A be an absolutely solid $*$-subalgebra in $LS(M)$ (that is if $x \in LS(M)$ and $y \in A$ are such that $|x| \leq |y|$, then $x \in A$) and let $M \subseteq A$. And let $d \in LS(M)$ be such that $[d, x] = dx - xd \in A$ for any $x \in A$. Then derivation $\delta(x) = [d, x]$ is inner on A.

Corollary. All spatial derivations on algebras $S(M)$ and $S(M,\tau)$ are inner.

Theorem 2. Let M be a properly infinite von Neumann algebra and let A be a $*$-subalgebra in $LS(M)$, $M \subseteq A$. Then any derivation δ on A is *Z-linear* (that is $\delta(z) = 0$ for any $z \in Z(A)$, when $Z(A)$ is the center of A).

[1] Muratov M.A., Chilin V.I. Algebras of measurable and locally measurable operators. – Kyiv, Pratsi In-ty matematiki NAN Ukraini. 2007. V. 69. – 390 p.(Russian).