Group classification and fundamental solutions of generalized linear Kolmogorov equations

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The class of (2+1)-dimensional generalized linear Kolmogorov equations

$$u_t - u_{xx} + A(x)u_y = 0,$$

where $u = u(t,x,y)$, $u_t = \frac{\partial u}{\partial t}$, $u_y = \frac{\partial u}{\partial y}$, and $u_{xx} = \frac{\partial^2 u}{\partial x^2}$ is considered. In the class under study the function $A = A(x)$ is an arbitrary smooth function of the independent variable x.

Our aim is to investigate the symmetry properties of equations from the class and to use them for construction of invariant fundamental solutions. The maximal algebras of invariance are found for the equations with non-trivial symmetry properties. By using the Berest–Aksenov algorithm [1, 2], the algebra of invariance of fundamental solutions of the linear Kolmogorov equation

$$u_t - u_{xx} + xu_y = 0,$$

is computed. The operators of the algebra obtained were used to construction of invariant fundamental solutions of the equation. It is shown that the fundamental solution

$$u = \frac{\sqrt{3}}{2\pi} \cdot \frac{\theta(t-t_0)}{(t-t_0)^2} \exp\left[-\frac{(x-x_0)^2}{4(t-t_0)} - \frac{3}{(t-t_0)^3}\left(y - y_0 - (t-t_0)\frac{x+x_0}{2} \right)^2 \right],$$

obtained by A.N. Kolmogorov [3] is a week invariant fundamental solution of the linear Kolmogorov equation.

References

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[3] A.N. Kolmogoroff, *Zufallige Bewegungen (Zur Theorie der Brownischen Bewegung)*, Ann. Math. **35**:2 (1934) 116–117.