On symmetry reduction of some classes of differential equations in the space $M(1,4) \times R(u)$

Volodymyr I. Fedorchuk

Let us consider a class of nonlinear five-dimensional d'Allembert equations of the form:

$$\Box_5 u = F(x_0, x_1, x_2, x_3, x_4, u, u_0, u_1, u_2, u_3, u_4) \qquad (1)$$

where $\Box_5 \equiv \frac{\partial^2}{\partial x_0^2} - \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_3^2} - \frac{\partial^2}{\partial x_4^2}$ is the d'Allembert operator in the five-dimensional Minkowski space $M(1,4)$, F is an arbitrary smooth function, $u = u(x_0, x_1, x_2, x_3, x_4)$, $u_\mu = \frac{\partial u}{\partial x_\mu}$, $\mu = 0, 1, 2, 3, 4$.

In the paper [1] the partial preliminary group classification of the class of equations of the form (1) has been performed. As the equivalence group of this class we have taken the generalized Poincaré group $P(1,4)$. The group $P(1,4)$ is a group of rotations and translations of the space $M(1,4)$. The group $P(1,4)$ is the smallest group, which contains as subgroups the extended Galilei group $G(1,3)$ [2] (symmetry group of the classical physics) and the Poincaré group $P(1,3)$ (symmetry groups of the relativistic physics).

The classes of obtained differential equations of the form (1), which are invariant with respect to nonconjugate subgroups of the group $P(1,4)$ can be written in the following form:

$$\Box_5 u = \Phi(J_1, J_2, ..., J_{t_1}), \qquad (2)$$

where Φ is an arbitrary smooth function, $\{J_1, J_2, ..., J_{t_1}\} (t_1 = 2, 3, ..., 10)$ are non-equivalent functional bases of the first-order differential invariants of nonconjugate subgroups of the group $P(1,4)$.

This report is devoted to the symmetry reduction of some of these classes. Until now, we have made the symmetry reduction of majority classes of equations (2) to classes of differential equations with a less number of independent variables.

References

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Pidstryhach IAPMM of the NAS of Ukraine,
Lviv, Ukraine
volfed@gmail.com