Multivariate sigma-functions to construct solutions for integrable systems

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This talk is devoted to the theory of multivariate σ -functions developed by V. Buchstaber, D. Leykin, V. Enolski (see [1–4]). The theory is based on series expansions, and has the advantage to be effective and easy for computation.

The first part of talk describes a construction of the series expansion of σ -function associated with a so called (n, s)-curve. As a by-product of the construction we obtain the basis of second kind differentials associated to the standard first kind differentials. The general scheme is illustrated by the examples of small genera.

Further we discuss on some applications and open problems related so called polylinear relations, namely, bilinear Hirota relations, which can be alternatively obtained from Klein's bidifferential formula; and trilinear relations, which produce addition formulas. We focus on the problem of regularization of the second kind integrals, which appears nontrivial in non-hyperelliptic case.

References

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