

Discontinues solutions to a quasilinear equation with variable

Valeriy Samoylenko and Yuliya Samoylenko

Department of Mathematical Physics, Kyiv National Taras Shevchenko
University, 64 Volodymyrs'ka Str., 01601 Kyiv, Ukraine

E-mail: vsam@univ.kiev.ua, yusam@univ.kiev.ua

Abstract

The talk deals with problem of finding discontinuous solutions [1, 2] to the equation

$$a_0(x)u_t + b_0(x)uu_x = 0, \quad (1)$$

where $a_0(x)$, $b_0(x)$ are infinitely differentiable functions, $x \in \mathbf{R}^1$, $t \in [0; T]$.

Equation (6) is generating equation for singularly perturbed one with variable coefficients [3, 4]

$$\varepsilon^2 u_{xxx} = a(x, \varepsilon)u_t + b(x, \varepsilon)uu_x, \quad (2)$$

where functions $a(x, \varepsilon)$, $b(x, \varepsilon)$ can be represented as

$$a(x, \varepsilon) = \sum_{k=0}^{\infty} a_k(x)\varepsilon^k, \quad b(x, \varepsilon) = \sum_{k=0}^{\infty} b_k(x)\varepsilon^k,$$

$t \in [0; T]$, $a_k(x)$, $b_k(x) \in C^{(\infty)}(\mathbf{R}^1)$, $k \geq 0$.

Using asymptotic one phase soliton type solution to the problem (2) the conditions of existing discontinuous solution to the problem (1) are proposed.

1. Hopf E. On the right weak solution of the Cauchy problem for a quasilinear equation of first order // Journ. of Math. and Mech. – 1969. – Vol. 19, N. 6. – P. 483–487.
2. Maslov V.P., Omelyanov G.A. Asymptotic soliton-type solutions with small dispersion // Uspekhi mat. nauk. – 1981. – V. 36 (219), N. 2. – P. 63 – 124.
3. Samoylenko Yul. Asymptotical expansions for one-phase soliton-type solution to perturbed Korteweg-de Vries equation // Proceedings of the Fifth International Conference “Symmetry in Nonlinear Mathematical Physics”. – K.: Institute of Mathematics. – 2004. – T. 3. – P. 1435 – 1441.
4. Samoylenko V.Hr., Samoylenko Yu.I. Asymptotic series for soliton type solutions to perturbed Korteweg-de Vries equation // Ukr. mat. journ. – 2005. – T.57, N 1. – P. 111–124.