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NONLOCAL INVARIANT REDUCTIONS OF THE SUPERSYMMETRIC BOUSSINESQ HIERARCHY

The supersymmetric Boussinesq hierarchy [1] on a functional supermanifold $M^{2|2} \subset C^\infty(\mathbb{S}^1 \times \Lambda_1; \Lambda_0^2 \times \Lambda_1^2)$ is considered in the form of Lax type evolution equations on the dual space to the Lie superalgebra of super-integro-differential operators of one anticommuting variable $\theta \in \Lambda_1$, which is the following:

$$dl/dt_i = [l, (l^{(3i+r)/3})_+], \quad i \in \mathbf{Z}_+, \quad r = 1, 2, \quad (1)$$

where $l = -D_\theta^6 + \phi D_\theta^3 - a D_\theta^2 + \chi D_\theta - b$, $(a, b, \phi, \chi)^T \in M^{2|2}$, $\Lambda := \Lambda_0 \oplus \Lambda_1$ is a Grassmann algebra over $\mathbb{C}$, $\Lambda_0 \supset \mathbb{C}$, $x \in \mathbb{S}^1 \simeq \mathbb{R}/2\pi\mathbb{Z}$, $D_\theta = \partial/\partial\theta + \theta\partial/\partial x$ is a superderivative and the subscript "+" denotes a pure differential part of the corresponding operator.

The complete integrability of the vector fields d/dt_i on their common invariant finite-dimensional supersubspace $M_N^{2|2} \subset M^{2|2}$:

$$M_N^{2|2} = \{(a, b, \phi, \chi)^T \in M^{2|2} : \text{grad } L_N[a, b, \phi, \chi] = 0\},$$
$$L_N = -\bar{\gamma}_1 + \sum_{j=1}^N c_j \lambda_j, \quad \bar{\gamma}_1 = \int_0^{2\pi} dx \int d\theta (a\chi + b\phi - (D_\theta \chi)\phi),$$

where $\bar{\gamma}_1$ is a local conservation law of the hierarchy (1), λ_j are some eigenvalues of the associated spectral problem, being invariant with respect to (1), and $c_j \in \Lambda_0$ for every $j = \overline{1, N}$, is studied by means of the differential-geometric structure of the supersubspace $M_N^{2|2}$. The existence of an even supersymplectic structure on $M_N^{2|2}$ and Hamiltonian representations for the reduced vector fields d/dt_i is established. It is shown also that the reduced evolution equations for the monodromy matrix of the associated spectral problem give Lax forms of the hierarchy (1) on $M_N^{2|2}$. To prove the Liouville type integrability [2] of the reduced vector fields d/dt_i a full set of their involutive conservation laws, generated by the supertraces of the squared and cubed monodromy matrices, is found.

References

- [1] Morosi C., Pizzocchero L., *osp(3, 2)* and *gl(3, 3)* supersymmetric KdV hierarchies, *Phys. Lett. A*, 1994, V. 185, 241–252.
- [2] Shander V.N., About complete integrability of ordinary differential equations on supermanifolds, *Funct. Analysis and its Appl.*, 1983, V. 17, N 1, 89–90. (in Russian)