Multicomponent solitons: effects of reductions

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To each simple Lie algebra $\mathfrak{g}$ one can relate a generalized Zakharov-Shabat system (GZSS):

$$L\psi(x,t,\lambda) \equiv i\frac{d\psi}{dx} + q(x,t) - \lambda J\psi(x,t,\lambda) = 0, \tag{1}$$

and a class of exactly solvable nonlinear evolution equations (NLEE) [1]. Here J is a constant element of the Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$ and the potential q is of the form $q(x,t) = [J, Q(x,t)]$. The direct and the inverse scattering problem for (1 for the class of smooth and vanishing potentials are equivalent to a Riemann-Hilbert problem on the real line [2, 3] if J is real; for complex-valued J one has to deal with a Riemann-Hilbert problem on a set of straight lines passing through the origin of the spectral λ-plane [4, 5].

We will outline the solution of the direct and inverse scattering problems for (1) in the case when $\mathbb{Z}_h$ reduction symmetry group [6] is imposed on it.

The soliton solutions of these NLEE will be constructed using an extension of the Zakharov-Shabat dressing method [7, 8]. A deeper analysis allows to classify the types of one-soliton solutions related to a given $\mathbb{Z}_h$-reduced NLEE.

The results are illustrated on NLEE related to $\mathfrak{g} \simeq sl(3)$, $so(5)$, $sp(4)$ and having well known applications in nonlinear optics, plasma physics etc.

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