Symmetry and Exact Solutions of the Rotating Shallow Water Equations for Spatial Shear Flows

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The rotating shallow water theory is a widely-used approximation for atmospheric and oceanic motions in the midlatitudes with relatively large length and time scales [1]. In Cartesian frame of reference the rotating shallow water equations for spatial shear flows have the following form

$$u_t + uu_x + vu_y + wu_z - fv + gh_x = 0, \quad v_t + uv_x + vv_y + wv_z + fu + gh_y = 0,$$
$$h_t + \left(\int_0^h u\,dz\right)_x + \left(\int_0^h v\,dz\right)_y = 0, \quad w = -\int_0^z (u_x + v_y)\,dz'. \tag{1}$$

Here (u, v, w) is the fluid velocity, h is the free surface height, f is constant Coriolis parameter and g is constant gravity acceleration.

We investigate symmetries and exact solutions of the model (1) using the group analysis theory [2], [3]. It is shown that equations (1) admit 9-dimensional Lie algebra L_9 of infinitesimal transformations. Lie algebra L_9 is proved to be isomorphic to the Lie algebra admitted by rotationless two-dimensional shallow water equations (or two-dimensional polytropic gas dynamic equations with $\gamma = 2$), which allows using its known optimal system of subalgebras. The invariant reductions of system (1) are given using representatives of the optimal system of subalgebras. Some of these subsystems are successfully integrated.

We construct and study wide classes of exact solutions, in particular, stationary rotationally-symmetric flows in a ring bounded by characteristics of system (1); time-dependent rotationally-symmetric fluid motions describing diffluence and collapse of a liquid ring; time-dependent fluid motions with closed trajectories.

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References

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