

Fields-currents duality and new hidden symmetry in non-abelian $O(N)$ invariant Goldstone-Higgs

Abstract

$O(N)$ invariant Goldstone model and Higgs sector of Standard Model has $\frac{N(N-1)}{2}$ usual Noethers currents: $j^{ab} = \phi^a \partial^{\leftrightarrow} \phi^b$. Infinite hierarchy of the currents given by Lie (Poisson) brackets for Noether currents are established. These vector currents are conserved upon action of the gravitational fields, but conservation is broken in Einstein-Cartan theory.

Essentially non-abelian structure of next family of trilinear currents is given by prepotentials of anticommutative gradients of fields and Noethers currents which satisfy Kirchhoff's law, that is that the current flowing in vertex geodetic triangle (square) of $O(N)$ sphere (the junctions of network) equals the currents flowing out. Prepotentials satisfy the Plücker relations for bivectors composed from currents.

The structure of currents for Standard Model has very nice geometric interpretation: $O(4)$ symmetry geodesic square gives as projection an tetraedre in vertices of which are the fields ϕ_i , $i=1,2,3,4$ and on links – the Noether currents. Plücker relations for prepotentials give the restriction on dimensionality of space-time: $d \geq 4$.

For $O(3)$ model there exists also dual (magnetic) topological current and similarity with 't Hooft-Polyakov monopole are pointed out. The analogy with hidden symmetry integrals of motion, Runge-Lenz vector for Kepler - Coulomb problem and second rank symmetric tensor for isotropic harmonic oscillator, is noted.