Symmetry Investigations of Certain Classes of Evolution Equations for Surface Morphology

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Surface processes play an important role in science and technical applications. One of the important topics in this direction is the deposition of particles from a two phase flow onto a surface. Examples for automotive applications are

- the deposition of soot on engine component parts like the valves,
- the deposition of soot in channels of particulate traps,
- the steaming of catalytic converters with wash-coat to get a special morphological structure.

One of the most important models describing such kind of process was derived by Kardar, Parisi and Zhang [1] and is of the form

$$\frac{\partial h(x,t)}{\partial t} = v_0 + \Delta h(x,t) + \frac{1}{2}\lambda(\nabla h(x,t))^2 + \eta(x,t) \tag{1}$$

A generalized model of this family of equations

$$\frac{\partial h(x,t)}{\partial t} + \alpha_1 \Delta h(x,t) + \alpha_2 \Delta^2 h(x,t) + \alpha_3 \Delta |\nabla h(x,t)|^2 - \alpha_4 |\nabla h(x,t)|^2 = 0 \tag{2}$$

was proposed in [2],where

$$\Delta h(x,t) = \frac{\partial^2 h(x,t)}{\partial x_1^2} + \frac{\partial^2 h(x,t)}{\partial x_2^2},$$

and

$$|\nabla h(x,t)| = \left[\left(\frac{\partial h(x,t)}{\partial x_1} \right)^2 + \left(\frac{\partial h(x,t)}{\partial x_1} \right)^2 \right]^{\frac{1}{2}}$$

are the two dimensional Laplacian and the amount of the gradient.

Another generalization was suggested by Hoppe, Litvinov, Linz [3] where we have

$$\frac{\partial h(x,t)}{\partial t} + \alpha_1 \Delta h(x,t) + \alpha_2 \Delta^2 h(x,t) + \Delta f_1(|\nabla h(x,t)|^2) + \\ f_2\left(|\nabla h(x,t)|^2\right) |\nabla h(x,t)|^2 = 0$$

with f_1 and f_2 arbitrary functions defined in $\mathbb{R}_+$.

In our presentation we show the application of symmetry investigations (described f. e. in [4, 5, 6, 7]) to these models in order to derive solutions. Our tools supporting the calculations are the two *Mathematica*-packages *MathLie* [7] and *MathLieAlg* [8].

References

[1] A.-L. Barabasi, H. E. Stanley, *Fractal Concepts in Surface Growth*, Cambridge University Press , 1995

[2] S. J. Linz, M. Raible, P. H"anggi, *Stochastic field equation for amorphous surface growth*, Lecture Notes in Physics 557, 473 - 483 (2000)

[3] R. H. W. Hoppe, W. G. Litvinov,, S. J. Linz, *On Solutions of Certain Classes of Evolution Equations for Surface Morphologies*, Nonlinear Phenomena in Complex Systems, 6, 1 (2003), 582 - 591

[4] N. H. Ibragimov, *CRC Handbook of Lie Group Analysis of Differential Equations Volumn 1, 1994, Volumn 2, 1995, Volumn 3, 1996*, CRC-Press, Boca Raton

[5] N. H. Ibragimov, *Elementary Lie Group Analysis and Ordinary Differential Equations*, John Wiley & Sons, Chichster 1999

[6] G. W. Bluman, J. D. Cole, *Similarity Methods for Differential Equations*, Springer, New York 1974

[7] G. Baumann, *Symmetry Analysis of Differential Equations*, Telos/Springer 2000

[8] R. Schmid, *Investigation of Lie Algebras*, private communication, Ulm 2002