Exact Solutions and Symmetry Operators for the Nonlocal Gross–Pitaevskii Equation with Quadratic Potential

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The nonlocal Gross–Pitaevskii equation (GPE) with quadratic potential in n-dimensional space is written as

$$\left(-i\hbar\partial_t + \hat{\mathcal{H}}(\vec{x},\hat{\vec{p}},t) + \int_{\mathbb{R}^n} V(t,\vec{x},\vec{y})|\Psi(\vec{y},t)|^2 d\vec{y}\right)\Psi(\vec{x},t) = 0, \qquad (1)$$

where $\vec{x},\vec{y}\in\mathbb{R}^n$, $\hat{\vec{p}} = -i\hbar\nabla_{\vec{x}}$, a linear operator $\hat{\mathcal{H}}$ and the nonlocal potential $V(t,\vec{x},\vec{y})$ are Weyl ordered smooth functions of $\hat{\vec{p}}$, $\vec{x}$, and $\vec{y}$. In the theory of Bose–Einstein condensate (BEC) Eq. (1) describes the condensate states Ψ taking into account the nonlocal interaction between atoms. The nonlocal BEC models are more realistic, and the nonlocal property can play the role of compensating factor to the collapse effect of the BEC states. In [1] we develop a method of semiclassical solutions for the Eq. (1) asymptotical in $\hbar$, $\hbar\to 0$, based on the WKB–Maslov theory of the complex germ. In the present work we construct the *exact* solution of the Cauchy problem for the Eq. (1) in the class of trajectory concentrated functions (TCF) when the linear operator $\hat{\mathcal{H}}$ and the nonlocal potential $V(t,\vec{x},\vec{y})$ are quadratic in $\hat{\vec{p}}$, $\vec{x}$, and $\vec{y}$. The nonlinear evolution operator is obtained in explicit form for the Eq. (1) in the class of TCF. With the help of symmetry operators, families of exact solutions of the equation are constructed. Exact expressions are obtained for the quasi-energies and their respective states. The Aharonov–Anandan geometric phases are found in explicit form for the quasi-energy states.

References

[1] Belov V.V., Trifonov A.Yu. and Shapovalov A.V., The Trajectory-Coherent Approximation and the System of Moments for the Hartree Type Equation, *IJMMS (USA)*, 2002, V.32, No 6, P. 325–370.

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