# Transverse Evolution Operator and Symmetry Operators for the Gross–Pitaevskii Equation in Semiclassical Approximation


**Alexey V. Borisov#, Alexander V. Shapovalov*# and Andrey Yu. Trifonov***

# *Theoretical Physics Department, Tomsk State University,*
*36 Lenin ave., Tomsk, 634050, Russia*
*e-mail: shpv@phys.tsu.ru*

* *Mathematical Physics Laboratory of the High Mathematics and Mathematical Physics Department, Tomsk Polytechnic University,*
*30 Lenin ave., Tomsk, 634050, Russia*
*e-mail: trifonov@mph.phtd.tpu.edu.ru*


The $n$ - dimensional Gross–Pitaevskii equation (GPE) with an attractive self-action reads

$$\left(-i\hbar\partial_t + \mathcal{H}(\hat{\vec{p}},\vec{x},t) - g^2|\Psi(\vec{x},t,\hbar)|^2\right)\Psi(\vec{x},t,\hbar) = 0, \tag{1}$$

where $\vec{x} \in \mathbb{R}^n$, $t \in \mathbb{R}^1$, $\hat{\vec{p}} = -i\hbar\nabla_{\vec{x}}$, $g$ is a real nonlinearity parameter, $\hbar$ is a small asymptotical parameter, $\hbar \to 0$; $\mathcal{H}(\hat{\vec{p}},\vec{x},t)$ is a linear operator quadratic in $\hat{\vec{p}}$ and Weyl ordered in $\hat{\vec{p}}$ and $\vec{x}$. The GPE (1) is one of the basic model equations in the theory of Bose–Einstein condensate (BEC). Localized solutions of the GPE describe the condensate in external electro-magnetic fields including fields of magnetic traps. In the context of the complex WKB–Maslov method a class of one-soliton trajectory concentrated functions (OSTCF) is introduced. Functions of this class are soliton-like fast-oscillating wave packets concentrated in a neighborhood of a trajectory in an effective phase space. A solution of the Cauchy problem is presented for the Eq. (1) in the class of OSTCF in semiclassical approximation. The evolution operator acting on the variables transversal on a wave packet vector is derived to obtain the leading term of the asymptotic solution in the class of OSTCF. A class of symmetry operators for the GPE is constructed in semiclassical approximation using the evolution operator. The three-dimensional GPE with the oscillator external field is considered as an illustration and the collapse problem is discussed.


**Acknowledgements.** The work has been supported in part by the Grant for Support of Russian Scientific Schools 1743.2003.2, and the Grant of the President of the Russian Federation YD-246.2003.02.