Anticipative analogues of diffusion processes

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Abstract. Linear stochastic differential equations with respect to the logarithmic process of smooth measure are studied. The theorem of existence and uniqueness is proved. Sufficient conditions for smoothness of solutions's trajectories are given.

This paper studies the properties of anticipative analogues of diffusion processes in non-Gaussian case.

It is well known that under sufficiently general assumptions, the one-dimensional diffusion process can be obtained as a solution of Ito stochastic differential equation

$$\begin{cases} dx(t) = a(x(t), t)dt + b(x(t), t)dw(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

Developed theory of stochastic differential equations essentially used the Ito construction of stochastic integral. For this reason the initial condition x_0 from (1) and Wiener process $\{w(t) : t \geq 0\}$ are independent. In attempt to refuse the requirement of independence we get the new class of problems. It connected with a fact that the solution $x(t)$, now, can not be adapted to a current of σ-algebras, which are generated by $w(t)$. In 70th this reason yielded [1,2] to the construction of extended stochastic integral w.r.t. Wiener process. The analogues of (1) with extended integral were considered by many authors [3 – 8]. Particularly it was observed that solutions of anticipative s.d.e. have some new properties not typical to the diffusion processes. For example, the solution of anticipative s.d.e. can possess discontinuous trajectories. Really, consider the following equation

$$\begin{cases} dx(t) = x(t)dw(t), t \in [0, 1] \\ x(0) = \operatorname{sign}(w(1)). \end{cases} \tag{2}$$

There the initial condition $x_0 = \operatorname{sign}(w(1))$ depends on $\{w(t), t \in [0, 1]\}$ and stochastic integral in (2) is Skorohod extended integral.

Then a solution $x(t)$ has a form:

$$x(t) = \text{sign}(w(1) - t)e^{w(t)-t/2}$$

and there is not any modification of $x(t)$ with continuous trajectories.

The aim of this paper is to consider a situation when extended stochastic integral with respect to the Wiener process is replaced by extended stochastic integral with respect to the logarithmic process of smooth measure.

Let us describe the following construction.

Consider a Banach space $X = C_0([0, 1])$ of continuous functions starting at zero and probability measure μ on Borel σ-algebra $\mathcal{B}(X)$. Suppose μ is quasiinvariant and Fomin differentiable [9] with respect to the directions of Hilbert space $H = L_2([0, 1])$ which is continuously and densely embedded to X by the inclusion map

$$i : H \ni f \mapsto \int_0^{\cdot} f(s)\, ds \in X.$$

It will be supposed that the logarithmic derivative λ of measure μ [9] satisfies the following condition

$$\forall p \geq 1\ \forall h \in H\ \ < \lambda, h > \in L_p(X, \mu).$$

Consider a space $\mathcal{F}C_b^\infty$ of cylindrical infinitedifferentiable functions

$$\mathcal{F}C_b^\infty = \{f = f(< e_1^*, \cdot >, \ldots, < e_n^*, \cdot >)|$$

$$n \in \mathbf{N}, f \in C_b^\infty(\mathbf{R}^n), e_k^* \in X^*\},$$

and finitedimensional smooth processes

$$\mathcal{F}C_b^\infty(H) = \{g = \sum_{k=1}^n g_k e_k|$$

$$n \in \mathbf{N}, g_k \in \mathcal{F}C_b^\infty, e_k \in H\}.$$

There we denote by $< \cdot, \cdot >$ the pairing between X^* and X, or H and H.

Definition 1. The stochastic derivative D,

$$D : W_p^1 \subset L_p(X, \mu) \to L_p(X, \mu, H)$$

is called the closure of a Frechet derivative ∇,

$$\nabla : \mathcal{F}C_b^\infty \subset L_p(X,\mu) \to L_p(X,\mu,H)$$

which is taken along the space H, i.e.

$$\nabla : f(< e_1^*, \cdot >, \ldots, < e_n^*, \cdot >) \mapsto \sum_{k=1}^{n} \frac{\partial f}{\partial x_k} i^*(e_k^*).$$

Particularly, if μ is a distribution of the process $\{\xi(t) : t \in [0,1]\}$ and $e_k^* = \delta_{t_k}$ is a unit measure concentrated in a point $t_k \in [0,1]$, then

$$D_t f(\xi(t_1), \ldots, \xi(t_n)) = \sum_{k=1}^{n} \frac{\partial f}{\partial x_k} \chi_{[0,t_k]}(t).$$

Definition 2. The operator I,

$$I : \mathcal{D}_q \subset L_q(X,\mu,H) \mapsto L_q(X,\mu)$$

which is adjoint to operator D,

$$D : W_p^1 \subset L_p(X,\mu) \mapsto L_p(X,\mu,H), \frac{1}{p} + \frac{1}{q} = 1$$

is called extended stochastic integral.

Remark 1. If μ is Wiener measure then I is Skorohod extended integral, which coincides with Ito integral on the space of non-anticipative processes.

Remark 2. The space $\mathcal{F}C_b^\infty(H)$ belongs to $\mathcal{D}_q, q \geq 1$, besides

$$I\left(\sum_{k=1}^{n} g_k e_k\right) =$$

$$= -\sum_{k=1}^{n} (< Dg_k, e_k > + g_k < \lambda, e_k >), \tag{3}$$

where $g_k \in \mathcal{F}C_b^\infty, e_k \in H$.

Remark 3. The random element φ belongs $\mathcal{D}_q, q \geq 1$ iff $\varphi \in L_q(X,\mu,H)$ and exists a random variable $g \in L_q(X,\mu)$ such that for all $f \in \mathcal{F}C_b^\infty(H)$ the formulae integration by parts holds:

$$\int_X < Df, \varphi > d\mu = \int_X f g \, d\mu \tag{4}$$

besides $g = I(\varphi)$,

Definition 3. The process $m(t) = I_{(\chi_{[0,t]})} = - < \lambda, \chi_{[0,t]} >$ is called the logarithmic process of measure μ.

The process $x \in L_1(X \times [0,1], d\mu \times dt)$ is said to be integrable w.r.t. logarithmic process if exists a random variable $g \in L_1(X, \mu)$ such that

$$\forall f \in \mathcal{F}C_b^\infty : \int\limits_X \left(\int\limits_0^1 x(t) D_t f dt \right) d\mu = \int\limits_X g f d\mu.$$

Let us denote g by $\int\limits_0^1 x(t) dm(t)$ and the space of integrable processes w.r.t $m(t)$ by $\mathcal{D}$, the integral $\int\limits_0^t x(s) dm(s)$ means $\int\limits_0^1 x(s) \chi_{[0,t]}(s) dm(s)$.

Remark. If $x \in \mathcal{D}$ such that $x \in L_q(X, \mu, H)$ and $\int\limits_0^1 x(s) dm(s) \in L_q(X, \mu)$ then $x \in \mathcal{D}_q$ and $Ix = \int\limits_0^1 x(s) dm(s)$.

If $x \in L_2([0,1])$ is non-random function then $\int\limits_0^1 x(s) dm(s)$ can be obtained analogously Ito construction.

If $x(t)$ is adapted process to a flow $\{\xi(t) : t \in [0,1]\}$, $x(t) = \sum\limits_{k=0}^{n-1} x_k \chi_{[t_k, t_{k+1}]}$, where $x_k \in L_\infty(X, \mu), 0 = t_0 < t_1 < \cdots < t_n = 1$, then

$$\int\limits_0^1 x(t) dm(t) = \sum_{k=0}^{n-1} x_k (m(t_{k+1}) - m(t_k)).$$

Example 1. If μ is a distribution of Wiener process $\{w(t) : t \in [0,1]\}$ then $m(t) = w(t)$ and $\int\limits_0^1 x(s) dw(s)$ is Skorohod extended integral, which coincides with Ito integral on the space of non-anticipative processes.

Example 2. $\boxed{\text{a}}$ Let $\mu_2(dx) = \rho(x)\mu_1(dx)$. Let us compare integrals I_{μ_1} and I_{μ_2}. If $\rho \in W_p^1(X, \mu_1)$ and $\rho > 0$ μ_1a.a. then

$$I_{\mu_2}(x) = I_{\mu_1}(x) - \left\langle \frac{D\rho}{\rho}, x \right\rangle, \tag{5}$$

if all variables in (5) has a proper moments.

$\boxed{\text{b}}$ If μ is a distribution of Wiener process $w(t)$,

$$\mu_1(dx) = \sqrt{2}e^{-\frac{w(1)^2}{2}}\mu(dx)$$

then $m_1(t) = w(t) + w(1)t$,

$$\int_0^t x(s)dm(s) = \int_0^t x(s)dw(s) + w(1)\int_0^t x(s)ds.$$

This example shows that a logarithmic process is not always a martingale. But in [10] is shown that $m(t)$ satisfies the following condition

$$E(m(t) - m(s)/\mathcal{F}_s) = 0,$$

where $0 \leq s \leq t, \mathcal{F}_s = \sigma(\xi(\tau)|\tau \leq s)$.

Consider the following linear s.d.e.

$$\begin{cases} dx(t) = a(t)x(t)dt + b(t)x(t)d\,m(t) \\ x(0) = \eta \end{cases} \tag{6}$$

where η is random variable; a, b are random processes.

Definition 4. The random process $x(t) \in L_1(X \times [0,1], d\mu \times dt)$ is a solution of (6) if

1) $x(t) \in L_1(X, \mu), t \in [0,1]$,
2) $a(t)x(t), b(t)x(t) \in L_1(X \times [0,1], d\mu \times dt)$
3) for all $t \in [0,1]$ we have

$$x(t) = \eta + \int_0^t a(s)x(s)ds + \int_0^t b(s)x(s)dm(s).$$

The condition 3) is equivalent to

3′) $\forall t \in [0,1], f \in \mathcal{F}C_b^\infty : \int_X \left(\int_0^t b(s)x(s)D_s f ds\right) d\mu =$

$$= \int_X \left(x(t) - \eta - \int_0^t a(s)x(s)ds\right) f d\mu. \tag{7}$$

Theorem 1 (existence and uniqueness). Let $b \in C([0,1])$ is non-random function; $a \in L_\infty(X \times [0,1], d\mu \times dt), \eta \in L_\infty(X, \mu)$. Then equation (6) has a unique solution. This solution is of the form

$$x(t) = \eta(A_t) \exp\left\{ \int_0^t a(s, T_s A_t) ds \right\} L_t, \tag{8}$$

where

$$T_t \omega = \omega + \int_0^{\cdot} \chi_{[0,t]}(z) b(z) dz, \quad \omega \in X,$$

$$A_t \omega = T_t^{-1} \omega = \omega - \int_0^{\cdot} \chi_{[0,t]}(z) b(z) dz,$$

$$L_t = \frac{d(\mu \circ T_t^{-1})}{d\mu}$$

is a density of shifted measure.

Example 3. Let $m(t)$ is a process from example 2b. Then the stochastic exponent

$$\begin{cases} d\mathcal{E}(s,t) = \mathcal{E}(s,t) dm(t), t \geq s \\ \mathcal{E}(s,s) = 1 \end{cases}$$

has a form

$$\mathcal{E}(s,t) = \exp\{m(t) - m(s) - \frac{1}{2}(t-s) - \frac{1}{2}(t-s)^2\}.$$

It is interesting to observe that $\mathcal{E}(s,t)$ has not a multiplicative property, i.e. for $s < \tau < t$ the equality

$$\mathcal{E}(s,t) = \mathcal{E}(s,\tau)\mathcal{E}(\tau,t)$$

does not hold

Example 4. Let $\{\xi_n : n \geq 0\}$ is a sequence of i.i.d random variable, $E\xi_n = 0$ and exists $\varepsilon > 0$ such that $E|\xi_n|^{2+\varepsilon} < \infty$. Suppose that distribution of ξ_n (in $\mathbf{R}^1$) is differentiable, $\lambda(x), x \in \mathbf{R}$ is a logarithmic derivative and $E(\lambda(\xi_n))^2 < \infty$.

Then random process $\xi(t) = \xi_0 t + \sum\limits_{n=1}^{\infty} \xi_n \frac{\sin n\pi t}{n}, t \in [0,1]$ has a continuous modification. The distribution of $\xi(t)$ is differentiable with respect to the directions of $i(L_2([0,1]))$, $i(f) = \int\limits_0^{\cdot} f(s)ds$. The logarithmic process $m(t)$ has a form

$$m(t) = \lambda(\xi_0)t + \sum_{n=1}^{\infty} \lambda(\xi_n)\frac{\sin n\pi t}{n}, t \in [0,1].$$

Suppose also that the distribution of ξ_n has a positive continuous density.

Then the equation

$$\begin{cases} dx(t) = x(t)dm(t) \\ x(0) = f(\xi(t_1), \dots, \xi(t_n)) \end{cases}$$

has a solution

$$x(t) = f\left(\xi(t_1) - \int\limits_0^{t\wedge t_1} ds, \dots, \xi(t_n) - \int\limits_0^{t\wedge t_n} ds\right) \cdot$$

$$\cdot \exp\left\{\int\limits_0^1 \lambda(\xi_0 - st)ds \cdot t + \sum_{n=1}^{\infty}\int\limits_0^1 \lambda\left(\xi_n - s\frac{\sin nt}{n}\right) ds\frac{\sin nt}{n}\right\}.$$

Formulae for density in terms of logarithmic derivative follows from [11].

Proof of theorem 1. Existence. Note that $\mu \circ T_t^{-1} \ll \mu$ because μ is quasiinvariant with respect to directions of H. Therefore L_t is well defined. Observe also that $x(t) \in L_1(X \times [0,1], d\mu \times dt)$.

For proof it suffices to verify formulae (7). Let $f \in \mathcal{F}C_b^{\infty}$ then

$$E\left(\int\limits_0^t b(s)x(s)D_s f ds\right) =$$

$$= E\left(\int\limits_0^t b(s)\eta(A_s)\exp\left\{\int\limits_0^s a(z, T_z A_s)dz\right\} L_s D_s f ds\right) =$$

$$= E\left(\int\limits_0^t b(s)\eta \exp\left\{\int\limits_0^s a(z, T_z)dz\right\} D_s f(T_s)ds\right). \quad (9)$$

Lemma 1 [4]. If $f \in \mathcal{F}C_b^{\infty}$ then for all $s \in [0,1]$:

$$\frac{d}{ds}f(T_s) = b(s) < Df, \chi_{[0,s]} > (T_s) = b(s)[D_s f](T_s)$$

$$\frac{d}{ds}f(A_s) = -b(s)[D_s f](A_s)$$

Using lemma 1 and integration by parts we will obtain the proof of existence.

Uniqueness. Let $x(t)$ is a solution of equation (6), $f \in \mathcal{F}C_b^\infty, t \in [0,1]$. It follows from (7) that

$$Ex(t)f(A_t) = E\eta f(A_t) + E\int\limits_0^t b(s)x(s)D_s(f(A_t))ds+$$

$$+E\int\limits_0^t a(s)x(s)D_s(f(A_t))ds. \tag{10}$$

Note that $f(A_t)$ belongs to $\mathcal{F}C_b^\infty$ (t is fixed) and (lemma 1)

$$f(A_t) = f(A_s) - \int\limits_s^t b(u)D_u(f(A_u))du. \tag{11}$$

Substituting (11)into (10), we get that

$$Ex(t)f(A_t) = E(\eta f) + E\int\limits_0^t a(s)x(s)f(A_s)ds,$$

i.e.

$$Ex(t,T_t)f\ \frac{d\mu\circ A_t^{-1}}{d\mu} =$$

$$= E(\eta f) + E\left(f\int\limits_0^t a(s,T_s)x(s,T_s)\frac{d\mu\circ A_t^{-1}}{d\mu}ds\right).$$

Since $f \in \mathcal{F}C_b^\infty$ is arbitrary, we can conclude that for every $t \in [0,1]$,

$$x(t,T_t)\cdot L_t^{-1} = \eta + \int\limits_0^t a(s,T_s)x(s,T_s)L_s^{-1}ds \quad (\text{mod } \mu),$$

where $L_t = \frac{d\mu\circ T_t^{-1}}{d\mu}$. Therefore $x(t,T_t)L_t^{-1} = \eta\exp\left\{\int\limits_0^t a(s,T_s)ds\right\}$ and $x(t)$ has a form (8). Theorem 1 is proved.

Remark. This method of proof was used in [4] in solving linear anticipating stochastic equations with respect to the Wiener process.

As follows from (8), for research of smoothness for solution's trajectories of s.d.e. (6) we need to study the trajectories of $\eta(A_t), t \in [0,1]$. The next theorem gives us sufficient conditions for continuity of $\eta(A_t)$ in terms of stochastic differentiability.

Theorem 2. Let there exists a modification of density $\rho(x,t) = \frac{d\mu\circ T_t^{-1}}{d\mu}(x), t \in [0,1]$ such that $\rho(x,t) > 0$ and continuous in t for all $x \in X$, then for each $\eta \in W_p^1$ there exists a process $\tilde{\eta}(x,t) = \eta(A_t(x)), t \in [0,1] \pmod{\mu}$ with absolutely continuous trajectories.

Proof. Consider a space $X \times [0,1]$ with measure $P(dx,dt) = \rho(x,t)d\mu(x)dt$. Define a derivative with respect to t,

$$\frac{\partial}{\partial t} : \mathcal{D}\left(\frac{\partial}{\partial t}\right) \subset L_1(X \times [0,1], P) \to L_1(X \times [0,1], P),$$

where $\mathcal{D}\left(\frac{\partial}{\partial t}\right)$ consists of functions f which are continuously differentiable with respect to t for all fixed $x \in X$, besides f is bounded with $\frac{\partial f}{\partial t}$.

Lemma 2. The operator $\frac{\partial}{\partial t}$ is closable.

Proof follows from the continuity $\rho(x,t)$ in t when x is fixed.

Lemma 3. The mapping j which transfer the random variable $\eta \in L_1(X,\mu)$ to the random process $\eta(A_t) \in L_1(X \times [0,1], P)$ is an isometry.

Lemma 4. The mapping $j : W_1^1 \to \mathcal{D}\left(\frac{\partial^c}{\partial t}\right)$ is continuous. There W_1^1 and $\mathcal{D}\left(\frac{\partial^c}{\partial t}\right)$ are provided with a norm

$$\|f\|_{W_1^1} = \|f\|_{L_1(X,\mu)} + \|Df\|_{L_1(X,\mu,H)},$$

$$\|g\|_{\mathcal{D}\left(\frac{\partial^c}{\partial t}\right)} = \|g\|_{L_1(X\times[0,1],P)} + \left\|\frac{\partial^c g}{\partial t}\right\|_{L_1(X\times[0,1],P)}.$$

Proof follows from lemmas 1 and 3.

Lemma 5. If ρ satisfies the conditions of the theorem 2, $f \in \mathcal{D}\left(\frac{\partial^c}{\partial t}\right)$ then exists $\tilde{f} = f \pmod{P}$ such that $\tilde{f}(x,t)$ is absolutely continuous in t for all $x \in X$.

To prove this, observe that for μ-a.a. $x \in X$ a function $f(x,\cdot)$ belongs to $W_1^1([0,1],dt)$, consequently exists such a modification $\tilde{f} = f \pmod{P}$ that is absolutely continuous in t for all $x \in X$.

The proof of the theorem 2 follows from lemma 2-5 .

As a corollary of the theorem 2 we can deduce the following theorem.

Theorem 3 (continuity of solution's trajectories). Let conditions of the theorems 1 and 2 be satisfied, $x(t)$ is a solution of (6) and $a_t \in W_p^1$ for all $t \in [0, 1]$.

Then there exists a process $\tilde{x}(t) = x(t) \pmod{\mu}, t \in [0, 1]$ which has continuous trajectories.

The next result shows that if stochastic derivative of random variable is essentially bounded then this random variable is "Lipschitz with respect to directions of H"

Definition 5. [12] A function $F : X \to \mathbf{R}$ is called H-Lipschitz with a constant $c > 0$ if for all $x \in X, h \in H$:

$$|F(x + i(h)) - F(x)| \leq c\|h\|_H.$$

A function $F : X \to \mathbf{R}$ is called H-Lipschitz μ-a.a. with a constant $c \geq 0$ if exists $\tilde{F} = F \pmod{\mu}$ such that $\tilde{F}$ is H-Lipschitz with a constant c.

Theorem 4. Let a measure μ satisfies the following condition.

There exists an orthonormal basis $\{i^*(e_n^*) : n \geq 1\}, e_n^* \in X^*$ of the space H such that

1) $(E^{\sigma_n} P_n \lambda - P_n \lambda) \in L_\infty(X, \mu, H)$,

$$\|E^{\sigma_n} P_n \lambda - P_n \lambda\|_{L_\infty(X,\mu,H)} \to 0, n \to \infty.$$

There E^{σ_n} is a mathematical expectation with respect to a σ-algebra, $\sigma_n = \sigma(< e_k^*, \cdot > |1 \leq k \leq n), P_n$ is ortoprojection on the

$$\mathcal{L}(i^*(e_k^*)|1 \leq k \leq n),$$

$$P_n \lambda = \sum_{k=1}^{n} < \lambda, i^*(e_k^*) > i^*(e_k^*).$$

2) A measure μ_n, the joint distribution of $< i^*(e_k^*), \cdot >,\ 1 \leq k \leq n$, has a continuous and positive density in $\mathbf{R}^n$.

Let $F \in W_p^1, p > 1$ and $DF \in L_\infty(X, \mu, H)$. Then F is H-Lipschitz μ-a.a. with a constant $\|DF\|_{L_\infty(X,\mu,H)}$.

For proof we need the following auxiliary lemma.

Lemma 6. [13] Let measure μ satisfies the condition 2) of the theorem 4, $F \in W_p^1, p > 1$.

Then $E^{\sigma_n}F \in W^1_{p-} = \bigcap_{\alpha<p} W^1_\alpha$ and the following formula holds

$$D(E^{\sigma_n}F) = E^{\sigma_n}(P_n DF) +$$

$$+ \langle P_n\lambda - E^{\sigma_n}P_n\lambda, E^{\sigma_n}F\rangle .$$

Proof of the theorem 4. Let $F \in L_\infty(X,\mu)$. There exists a Borel function $\varphi_n : \mathbf{R}^n \to \mathbf{R}$ such that

$$F_n = \varphi_n(< e_1^*, \cdot >, \dots, < e_n^*, \cdot >) \quad (\mathrm{mod}\ \mu).$$

It follows from lemma 6 that $\varphi_n \in W^1_p(\mathbf{R}^n, \mu_n)$, consequently exists $\tilde{\varphi}_n = \varphi_n$ $(\mathrm{mod}\ \mu_n)$ which is Lipschitz with a constant

$$\|DF\|_{L_\infty(X,\mu,H)} + \|F\|_{L_\infty(X,\mu)} \cdot \|E^{\sigma_n}P_n\lambda - P_n\lambda\|_{L_\infty(X,\mu,H)}.$$

Therefore F_n is H-Lipschitz μ-a.a. with the same constant. An approximation of function F by $E^{\sigma_n}F$ completes a proof in a case when $F \in L_\infty(X,\mu)$.

The general case reduces to $F \in L_\infty(X,\mu)$ by the approximation $F = \lim\limits_{n\to\infty} \psi_n(F)$, where

$$\psi_n = \begin{cases} t, |t| \le n \\ n \cdot \operatorname{sign} t, |t| > n. \end{cases}$$

Remark 1. This idea of the proof was realized in Wiener case in [12].

Remark 2. Wiener measure and the measure of example 4 are satisfied the conditions of theorem 4. If we take in this case $i^*(e_n^*) = \sqrt{2}\cos n\pi t$ then $E^{\sigma_n}P_n\lambda - P_n\lambda = 0$.

Theorem 5. (Holder condition for solutions's trajectories). Let conditions of theorem 1 and 4 are fulfilled and $\eta \in W^1_p, a(t) \in W^1_p, t \in [0,1]$,

$$D\eta \in L_\infty(X,\mu,L_\infty[0,1]),$$

$$\int_0^1 \|Da(t)\|_{L_\infty(X,\mu,L_\infty[0,1])} dt < \infty.$$

Suppose there exists $\alpha > 0$ such that for all $c > 0$

$$\sup_{\|h\|_H \le c} E\left|\frac{d\mu(\cdot - i(h))}{d\mu}\right|^{1+\alpha} < \infty.$$

Then there exists a modification of the solution (8) of the equation (6) with trajectories satisfying Holder condition with an index ρ, for every $\rho < \frac{1}{2}$.

Holder condition for L_t follows from Kolmogorov's theorem, the formulae for the density of shifted measure in terms of logarithmic derivative[11] and assumption, that $< \lambda, h > \in L_p(X, \mu)$ for all $h \in H, p > 1$. Application of the theorem 4 completes the proof.

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