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I would like to propose one more approach to well-known general problem of S.Kakutani and J.Doob of unification both Martingale convergence and Ergodic theorems.

Suppose that (Ω, F, λ) is a probability space, T is its automorphism, $f \in L_1(\Omega)$. Let $\{F_n\}_{1\le n\le\infty}$ be a monotone sequence of σ-subalgebras F such that $F_n \uparrow F_\infty$ (or $F_n \downarrow F_\infty$) as $n \to \infty$. Set

$$A_n f = \frac{1}{n}\sum_{k=0}^{n} f \circ T^k, \quad f^* = \lim_{n\to\infty} A_n f, \quad f^*_\infty = \mathrm{E}(f^*|F_\infty).$$

Problem. *To prove convergence a.e. of conditional expectations* $\mathrm{E}(A_n f|F_n) \to f^*_\infty$ *as* $n \to \infty$.

Comments. In the degenerate case $F_n \equiv F$ it will be $\mathrm{E}(A_n f|F_n) = A_n f$, and we have usial Ergodic theorem. In the case $T \equiv \mathrm{id}$ it will be $\mathrm{E}(A_n f|F_n) = \mathrm{E}(f|F_n)$, and we have Martingale convergence theorem for reversed martingale (if $F_n \downarrow$) or for regular strightforward martingale (if $F_n \uparrow$).

Known. It is proved already (Math. Notes, 1998, 64:2, P.266–269) that proposed statement holds for all $f \in L \log L$. Also, it is proved convergence in L_p for every $p \in [1, \infty)$ (Ibid).

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