A NOTE ON CLASSIFICATION OF HOLOMORPHIS MATRICES UP TO SIMILARITY

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V. I. Arnol'd [1] developed a normal form for matrices $A(x)$ holomorphic at $x = 0$ relative to a holomorphic similarity transformation $S(x)A(x)S^{-1}(x)$, where $S(x)$, $S(x)^{-1}$ are matrices holomorphic at 0. This form is not canonical: matrices in normal form may be different but holomorphically similar.

We shall show that the problem of the canonical form of even the first two matrices in the expansion $A(x) = A_0 + xA_1 + \ldots$ is wild, i.e., contains the classical unsolved problem of classifying a pair of linear operators in a finite-dimensional vector space; consequently (see [2]), it also contains the problem of classifying any sequence of operators.

THEOREM. Two matrices in Arnol'd normal form

$$A(x) = \begin{pmatrix} 0 & E & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & xM & 0 & xN \\ xE & 0 & 0 & 0 \end{pmatrix}, \quad B(x) = \begin{pmatrix} 0 & E & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & xM' & 0 & xN' \\ xE & 0 & 0 & 0 \end{pmatrix}$$

(all blocks are square, M, N, M', N' are constant matrices) are holomorphically similar if and only if there exists a constant matrix C such that $CMC^{-1} = M'$, $CNC^{-1} = N'$.

Proof. Express $S(x) = S_0 + xS_1 + \ldots$ as a 4×4 block matrix and equate the coefficients of x^0, x^1 in the equality $S(x)A(x) = B(x)S(x)$.

LITERATURE CITED

1. V. I. Arnol'd, Usp. Mat. Nauk, 26, No. 2, 101-114 (1971).
2. I. M. Gel'fand and V. A. Ponomarev, Funkts. Anal. Prilozhen., 13, No. 4, 81-82 (1969).

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