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WEIGHTED ESTIMATION AND REDUCTION OF THE INFLUENCE OF BOUNDED PERTURBATIONS IN DESCRIPTOR CONTROL SYSTEMS

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For a class of linear descriptor systems, we establish new criteria for the existence of the regularities of control guaranteeing the asymptotic stability and satisfying the required estimate for the weighted level of decay of bounded disturbances. We propose a procedure of generalized H_∞-optimization of the descriptor systems with controlled and observed outputs. The main computational procedures of the suggested algorithm are reduced to the solution of linear and quadratic matrix inequalities with additional rank constraints. We also present an example of descriptor control system intended for the stabilization of an electric circuit.

1. Introduction

Descriptor control systems are encountered in the design and investigation of the dynamics of complex objects in mechanics, electric engineering, economics, etc. (see, e.g., [1–6]). In the construction of the equations of motion for objects of this kind in terms of variables that describe actual physical processes, it is necessary to take into account not only differential but also algebraic relations and constraints in the phase space. For this reason, descriptor systems are also called differential-algebraic or singular systems. Available methods for the construction and investigation of solutions for the class of linear descriptor systems are based on the application of the theory of canonical forms of matrix pencils and generalized inverse matrices [2, 7].

As modern directions of investigations in the control theory of both ordinary and descriptor systems, we can mention the methods of robust stabilization and H_2/H_∞-optimization guaranteeing the robust stability of equilibrium states and minimization of the negative influence of external disturbances on the dynamics of controlled objects. The role of a typical performance criterion in the problems of H_∞-optimization of continuous and discrete systems with trivial initial state is the level of damping of the external disturbances corresponding to the maximal value of the ratio of L_2-norms of the vectors of controlled output of the object and disturbances (see, e.g., [8–11]). More general performance criteria characterizing the weighted level of damping of external and initial disturbances caused by the nonzero initial vector were used in [12–17]. By using the weighting coefficients appearing in these performance criteria, we can establish priorities in the collection of components of the controlled output and unknown disturbances in control systems. Moreover, the role of components of the unknown disturbances can be played both by the external disturbances acting upon the system and by the errors of measured output.

The available methods of synthesis of the H_∞-control are based on the criteria of validity of the upper bounds for the corresponding performance criteria established in terms of matrix equations and linear matrix inequalities [8, 18, 19]. For the class of linear descriptor systems, similar statements were established in [20–23]. For the known methods of H_∞-optimization of these systems, see, e.g., [5, 20, 22, 24, 25].

In the present paper, we continue our investigations originated in [16, 17] and devoted to the problems of synthesis of generalized H_∞-control for linear descriptor systems. We propose new criteria for the existence and

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algorithms of construction of static and dynamic regulators guaranteeing the required estimate for the weighted level of influence of bounded disturbances on the quality of transient processes in descriptor systems with controlled and observed outputs. The practical realization of these algorithms is reduced to the solution of linear and quadratic matrix inequalities under additional rank restrictions. A typical specific feature of the obtained results as compared with the known facts is connected with the use of weighted performance criteria, which enables one to find the required characteristics of descriptor control systems. In Sec. 2, we also present a procedure of construction of the worst disturbance and the worst initial vector for the weighted performance criteria. Our main statements are formulated in Sec. 3 without additional restrictions imposed on the matrix coefficients of the controlled system and its outputs, which were used in numerous works (see, e.g., [20, 22]).

We use the following notation:

I_n is the identity matrix of order n;

$0_{n\times m}$ is the $n \times m$ null matrix;

$X = X^\top > 0\ (\geq 0)$ is a positive-definite (nonnegative-definite) matrix X;

$\sigma(A)\ (\rho(A))$ is the spectrum (spectral radius) of a matrix A;

$A^{-1}\ (A^{+})$ is the inverse (pseudoinverse) matrix;

$\operatorname{Ker} A$ is the kernel of the matrix A;

W_A is a matrix whose columns form a basis in the kernel of matrix A;

$\operatorname{Co}\{A_1, \dots, A_\nu\}$ is a convex polyhedron (polytope) with vertices $A_1, \dots, A_\nu$ in the space of matrices;

$\|x\|$ is the Euclidean norm of the vector x,

and

$\|w\|_P = \left(\int_0^\infty w^\top P w \, dt \right)^{\frac{1}{2}}$ is the weighted L_2-norm of a vector function $w(t)$.

2. Admissible Descriptor Systems with Disturbances

Consider a linear descriptor system

$$E\dot{x} = Ax + Bw, \qquad z = Cx + Dw, \quad x(0) = x_0, \tag{2.1}$$

where x_0 is the initial vector and $x \in \mathbb{R}^n$, $w \in \mathbb{R}^s$, and $z \in \mathbb{R}^k$ are, respectively, the vectors of state, external disturbances, and output. All matrix coefficients in (2.1) are constant. Moreover, the matrix pencil $F(\lambda) = A - \lambda E$ is *regular*, i.e., $\det F(\lambda) \not\equiv 0\ (\lambda \in \mathbb{C})$. In the case where $\rho = \operatorname{rank} E < n$, system (2.1) can be rewritten in the form

$$\dot{x}_1 = A_1 x_1 + B_1 w, \qquad N\dot{x}_2 = x_2 + B_2 w, \qquad z = C_1 x_1 + C_2 x_2 + Dw, \tag{2.2}$$

where

$$x = R \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \qquad x_0 = R \begin{bmatrix} x_{01} \\ x_{02} \end{bmatrix}, \qquad LB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad CR = \begin{bmatrix} C_1, C_2 \end{bmatrix},$$

$x_1 \in \mathbb{R}^r$, $x_2 \in \mathbb{R}^{n-r}$, and L and R are nonsingular matrices of reduction of the pair (A, E) to the canonical

Weierstrass form [7]:

$$LAR = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}, \qquad LER = \begin{bmatrix} I_r & 0 \\ 0 & N \end{bmatrix}. \tag{2.3}$$

The eigenvalues of the matrix A_1 form a finite spectrum $\sigma(F) = \{\lambda_1, \ldots, \lambda_r\}$ and N is a nilpotent matrix of index ν. The first subsystem in (2.2) is dynamical and the second subsystem is algebraic. Moreover, for $\nu > 1$, its solution contains impulsive components [5]. The matrix pencil $F(\lambda)$ is called *stable* and *not impulsive* if $\sigma(F) \subset \{\lambda\colon \operatorname{Re}\lambda < 0\}$ and $N = 0$, respectively. The descriptor system (2.1) is called *admissible* if the corresponding matrix pencil $F(\lambda)$ is regular, stable, and not impulsive. It is convenient to describe the introduced properties of matrix pencils via the corresponding pairs of matrices (E, A). In particular, as a criterion of absence of impulsive modes in system (2.1), we can use the following condition [3]:

$$\operatorname{rank} \begin{bmatrix} E & 0 \\ A & E \end{bmatrix} = n + \rho.$$

Lemma 2.1 [16]. *A pair of matrices (E, A) is not impulsive if and only if the system of matrix equations*

$$AZE = EZA, \qquad Z = ZEZ, \qquad E = EZE$$

is consistent with respect to Z.

Lemma 2.2 [22]. *System (2.1) is admissible if and only if the system of relations*

$$A^\top X + X^\top A < 0, \qquad E^\top X = X^\top E \geq 0$$

is consistent with respect to X.

Assume that the vector of disturbances $w(t)$ in system (2.1) is bounded in the weighted L_2-norm $\|w\|_P$. For this system, we introduce the following performance criterion [13]:

$$J = \sup_{(w,x_0)\in\mathcal{W}} \frac{\|z\|_Q}{\sqrt{\|w\|_P^2 + x_0^\top X_0 x_0}}, \tag{2.4}$$

where $\mathcal{W}$ is the set of pairs (w, x_0) for which system (2.1) has a solution and the inequality

$$\|w\|_P^2 + x_0^\top X_0 x_0 \neq 0$$

is true, $P = P^\top > 0$, $Q = Q^\top > 0$, and $X_0 = E^\top H E \geq 0$ are weight matrices $(H = H^\top > 0)$. The value of J characterizes the weighted level of the influence of external and initial disturbances on the output of system (2.1). By applying the weighting matrix coefficients P, Q, and X_0 in (2.4), we can establish priorities in the influence of components of the vectors w, z, and x_0 on the performance criterion J. It is reasonable to use this possibility, e.g., in the case where the components of the vector w are not only external disturbances but also measurement errors in the output of the system (see, e.g., [15]).

For $x_0 \in \operatorname{Ker} E$, we denote expression (2.4) by J_0. It is clear that $J_0 \leq J$. For the identity matrices $P = I_s$ and $Q = I_k$, the expression J_0 is a typical performance criterion used in the problems of H_∞-optimization of systems and its value coincides with the H_∞-norm of the matrix transfer function [10]:

$$\|G_{zw}\|_\infty = \sup_{\omega\in\mathbb{R}} \sqrt{\lambda_{\max}(G^\top(-i\omega)G(i\omega))}, \qquad G(\lambda) = C(\lambda E - A)^{-1}B + D.$$

In this case, all components of the vectors of disturbances and output of the system exert equivalent influence on the value of the performance criterion J_0.

The vector of disturbances $w(t)$ and the initial vector x_0 are called the *worst* vectors in system (2.1) for the performance criterion J if the supremum of (2.4), i.e.,

$$\|z\|_Q^2 = J^2\big(\|w\|_P^2 + x_0^\top X_0 x_0\big),$$

is attained on the values of these vectors. Methods for the determination of these vectors in some special cases were proposed in [12, 15].

In the class of admissible systems (2.1), we establish necessary and sufficient conditions for the attainment of the upper bounds $J_0 < \gamma$ and $J < \gamma$ for given $\gamma > 0$.

Lemma 2.3 [23]. *If there exist matrices X and $S = S^\top \geq 0$ satisfying the system of linear matrix inequalities*

$$\begin{bmatrix} S & S - E^\top X \\ S - X^\top E & 0 \end{bmatrix} \geq 0, \tag{2.5}$$

$$\Psi(X) = \begin{bmatrix} A^\top X + X^\top A + C^\top QC & X^\top B + C^\top QD \\ B^\top X + D^\top QC & D^\top QD - \gamma^2 P \end{bmatrix} < 0, \tag{2.6}$$

then system (2.1) is admissible and $J_0 < \gamma$. The converse assertion is true provided that

$$\operatorname{rank} \begin{bmatrix} E \\ D^\top QC \end{bmatrix} = \rho. \tag{2.7}$$

Condition (2.5) means that $S = E^\top X = X^\top E \geq 0$. It is possible to show that the linear matrix inequality (2.6) is consistent for X if and only if

$$D^\top QD < \gamma^2 P, \qquad D_1^\top QD_1 < \gamma^2 P,$$

where $D_1 = D - CA^{-1}B$. It follows from these relations and Lemma 2.3 that $J_0 > \gamma_0$, where

$$\gamma_0 = \max\big\{\gamma\colon\ \det\big[(D^\top QD - \gamma^2 P)(D_1^\top QD_1 - \gamma^2 P)\big] = 0\big\}.$$

Lemma 2.4 [23]. *If the system of relations (2.5), (2.6) is consistent and*

$$S \leq \gamma^2 X_0, \qquad \operatorname{rank}(S - \gamma^2 X_0) = \rho, \tag{2.8}$$

then system (2.1) is admissible and $J < \gamma$. The converse assertion is true under condition (2.7).

Under the conditions of Lemmas 2.3 and 2.4, the null state of system (2.1) with structured indeterminacy of the vector of disturbances

$$w = \frac{1}{\gamma}\Theta z, \qquad \Theta^\top P \Theta \le Q,$$

is robustly stable with common Lyapunov function $v(x) = x^\top S x$. This assertion follows from transformation (2.3) and the theorem on robust stabilization of the linear system [13] (Theorem 3.3.1).

By using Lemmas 2.3 and 2.4, we obtain algorithms for finding the performance criteria J_0 and J for system (2.1) on the basis of the solutions of the corresponding optimization problems. In particular, under the conditions of Lemma 2.4, we find

$$J = \inf\left\{\gamma\colon\ \Psi(X) < 0,\ \ 0 \le E^\top X = X^\top E \le \gamma^2 X_0\right\}.$$

Lemma 2.5. *Suppose that system (2.1) is admissible and that the linear matrix inequalities (2.5) and (2.8) and the following equality are true for some matrices* X *and* $S = S^\top \ge 0$*:*

$$A_1^\top X + X^\top A_1 + X^\top R_1 X + Q_1 = 0, \tag{2.9}$$

where

$$A_1 = A + BR^{-1}D^\top QC, \qquad R_1 = BR^{-1}B^\top, \qquad Q_1 = C^\top\left(Q + QDR^{-1}D^\top Q\right)C,$$

$$R = \gamma^2 P - D^\top Q D > 0, \qquad \textit{and} \qquad \gamma = J.$$

Then the structured vector of external disturbances in the form of a linear feedback by the state

$$w = K_0 x, \qquad K_0 = R^{-1}(B^\top X + D^\top Q C), \tag{2.10}$$

and an arbitrary initial vector $x_0 \in \operatorname{Ker}\ (S - J^2 X_0)$ *are the worst vectors for the performance criterion* J *for system (2.1).*

Proof. If $S = E^\top X = X^\top E \le \gamma^2 X_0$ and $\Psi(X) \le 0$, then

$$\dot v(x) + z^\top Q z - \gamma^2 w^\top P w = \left[x^\top, w^\top\right] \Psi(X) \begin{bmatrix} x \\ w \end{bmatrix} \le 0, \tag{2.11}$$

where $\dot v(x)$ is the derivative of the function $v(x) = x^\top S x$ by system (2.1). Integrating this expression from 0 to τ and taking into account the fact that $v(x(\tau)) \to 0$ as $\tau \to \infty$, we obtain

$$\|z\|_Q^2 - \gamma^2 \|w\|_P^2 \le x_0^\top S x_0 \le \gamma^2 x_0^\top X_0 x_0, \tag{2.12}$$

i.e., $J \le \gamma$. The validity of both equalities in (2.12) means that $\gamma = J$ and, hence, the corresponding disturbance vector $w(t)$ and the initial vector x_0 are the worst vectors for the performance criterion J.

It is easy to see that the first equality in (2.12) is true if the structure of the disturbance vector w is described by (2.10), where X is a solution of the matrix Riccati equation (2.9) and $x(t)$ is a solution of the system

$$E\dot x = (A + BK_0)x, \qquad x(0) = x_0.$$

Moreover, the right-hand side of relation (2.11) is equal to zero. The second equality in (2.12) is true for

$$x_0^\top (S - \gamma^2 X_0) x_0 = 0.$$

Under the condition $S \leq \gamma^2 X_0$, the last equality means that $x_0 \in \operatorname{Ker}(S - \gamma^2 X_0)$.

The lemma is proved.

3. Linear Systems with Controlled and Observed Outputs

Consider a control system

$$\begin{aligned} E\dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0, \\ z &= C_1 x + D_{11} w + D_{12} u, \\ y &= C_2 x + D_{21} w + D_{22} u, \end{aligned} \tag{3.1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $w \in \mathbb{R}^s$, $z \in \mathbb{R}^k$, and $y \in \mathbb{R}^l$ are the vectors of state, control, external disturbances, and controlled and observed outputs, respectively, and all matrix coefficients of the corresponding dimensions are constant. Moreover, the pair (E, A) is regular and $\operatorname{rank} E = \rho \leq n$. We are interested in the regularities of control guaranteeing the required estimates for the performance criteria of type (2.4) for a closed system with respect to the vector of controlled output z. The static and dynamic regulators minimizing the performance criterion J are called J*-optimal*. The J_0-optimal control for the identity weight matrices P and Q is called H_∞*-optimal*.

In the investigation of the class of systems (3.1), it is customary to use their properties of C-, R,- and I*-controllability*, as well as the dual properties of their C-, R,- I*-observability* [5, 24]. In particular, for the solvability of the generalized problems of H_∞-optimization, the triple of matrices (E, A, B_2) must be stabilized and I-controlled. This is equivalent to the requirement of existence of a matrix K such that the pair of matrices $(E, A + B_2 K)$ is stable and not impulsive, i.e., admissible. The roles of criteria of I-controllability of the triple (E, A, B_2) and I-observability of the triple (E, A, C_2) are played by the equalities [26]

$$\operatorname{rank}\begin{bmatrix} E & 0 & 0 \\ A & E & B_2 \end{bmatrix} = n + \rho \qquad \text{and} \qquad \operatorname{rank}\begin{bmatrix} E & A \\ 0 & E \\ 0 & C_2 \end{bmatrix} = n + \rho.$$

3.1. Static Regulator. In the case where we use a static regulator of the observed output

$$u = Ky, \qquad \det(I_m - K D_{22}) \neq 0, \tag{3.2}$$

where K is the required matrix of gain factors, the closed system takes the form

$$E\dot{x} = A_* x + B_* w, \qquad z = C_* x + D_* w, \quad x(0) = x_0, \tag{3.3}$$

where

$$A_* = A + B_2 K_* C_2, \qquad B_* = B_1 + B_2 K_* D_{21}, \qquad C_* = C_1 + D_{12} K_* C_2,$$

$$D_* = D_{11} + D_{12} K_* D_{21}, \qquad K_* = (I_m - K D_{22})^{-1} K.$$

In order to get the required estimate $J < \gamma$, we use Lemma 2.4. We represent condition (2.6) for system (3.3) in the form of a quadratic matrix inequality as follows:

$$W + U^\top K_* V + V^\top K_*^\top U + V^\top K_*^\top R K_* V < 0, \tag{3.4}$$

where

$$W = \begin{bmatrix} A^\top X + X^\top A + C_1^\top Q C_1 & X^\top B_1 + C_1^\top Q D_{11} \\ B_1^\top X + D_{11}^\top Q C_1 & D_{11}^\top Q D_{11} - \gamma^2 P \end{bmatrix},$$

$$U = \big[B_2^\top X + D_{12}^\top Q C_1,\ D_{12}^\top Q D_{11}\big], \qquad V = \big[C_2,\ D_{21}\big], \qquad R = D_{12}^\top Q D_{12} \geq 0.$$

We formulate a criterion of consistency for this inequality under the conditions that $m < q$ and $l < q$ $(q = n + s)$, which are natural for system (3.1).

Lemma 3.1. *The matrix inequality (3.4) possesses a solution* $K_* \in \mathbb{R}^{m \times l}$ *if and only if:*

(a) $W_V^\top W W_V < 0$

and one of the following conditions is satisfied:

(b) $R = 0,\ W_U^\top W W_U < 0;$

(c) $R > 0,\ W < U^\top R^{-1} U;$

(d) $R \geq 0,\ 1 \leq \operatorname{rank} R < m,\ W_{U_0}^\top \big(W - U^\top R^+ U\big) W_{U_0} < 0,\ U_0 = W_R^\top U.$

Proof. We now show that the proof of the indicated criteria of existence of the solution K_* of the quadratic matrix inequality (3.4) for $R \geq 0$ is reduced to the application of the known necessary and sufficient consistency conditions for linear matrix inequalities (the projection lemma [18]). For $R = 0$, the matrix inequality (3.4) is linear and the criterion of its consistency is given by conditions (a) and (b). For $R > 0$, by the Schur lemma, the matrix inequality (3.4) can be rewritten in the form of a linear matrix inequality (see, e.g., [8, p. 8]):

$$\begin{bmatrix} W + U^\top K_* V + V^\top K_*^\top U & V^\top K_*^\top \\ K_* V & -R^{-1} \end{bmatrix} < 0 \qquad \text{or} \qquad \widehat{W} + \widehat{U}^\top K_* \widehat{V} + \widehat{V}^\top K_*^\top \widehat{U} < 0,$$

where

$$\widehat{W} = \begin{bmatrix} W & 0 \\ 0 & -R^{-1} \end{bmatrix}, \qquad \widehat{U} = \big[U \quad I_m\big], \qquad \widehat{V} = \big[V \quad 0_{l \times m}\big].$$

The necessary and sufficient conditions for the consistency of the last inequality are the inequalities

$$W_{\widehat{U}}^\top \widehat{W} W_{\widehat{U}} < 0 \quad \text{and} \quad W_{\widehat{V}}^\top \widehat{W} W_{\widehat{V}} < 0,$$

i.e., the conditions (a) and (c), because

$$W_{\widehat{U}} = \begin{bmatrix} I_q \\ -U \end{bmatrix} \qquad \text{and} \qquad W_{\widehat{V}} = \begin{bmatrix} W_V & 0 \\ 0 & I_m \end{bmatrix}.$$

Now let $R = LL^\top \geq 0,\ L \in \mathbb{R}^{m\times r}$, and $r = \operatorname{rank} R = \operatorname{rank} L < m$. Without loss of generality, we seek the solution of (3.4) in the form

$$K_* = L^{+\top} K_1 + L^\perp K_2, \qquad L^+ = (L^\top L)^{-1} L^\top, \qquad L^\perp = W_{L^\top} = W_R \in \mathbb{R}^{m\times m-r},$$

where $K_1 \in \mathbb{R}^{r\times l}$ and $K_2 \in \mathbb{R}^{m-r\times l}$ are unknown matrices. By using the equality $K_*^\top R K_* = K_1^\top K_1$, we get the following linear matrix inequality for K_1:

$$W_1 + U_1^\top K_1 V + V^\top K_1^\top U_1 + V^\top K_1^\top K_1 V < 0,$$

where

$$W_1 = W + U_0^\top K_2 V + V^\top K_2^\top U_0 \qquad \text{and} \qquad U_1 = L^+ U.$$

As shown above, the consistency criterion for this inequality has the form of inequalities

$$W_1 < U_1^\top U_1 \qquad \text{and} \qquad W_V^\top W_1 W_V < 0,$$

i.e.,

$$W - U^\top R^+ U + U_0^\top K_2 V + V^\top K_2^\top U_0 < 0 \qquad \text{and} \qquad W_V^\top W W_V < 0.$$

By using the consistency criterion for the first linear matrix inequality for K_2 once again, we arrive at the conditions (a) and (d).

The lemma is proved.

Thus, by virtue of Lemmas 2.4 and 3.1, we get the following assertion:

Theorem 3.1. *Suppose that the conditions*

$$R_0 = D_{12}^\top Q D_{12} > 0, \qquad R_1 = \gamma^2 P - D_{11}^\top Q_1 D_{11} > 0, \qquad Q_1 = Q - Q D_{12} R_0^{-1} D_{12}^\top Q \tag{3.5}$$

are satisfied and the system of relations (2.5), (2.8), and

$$W_V^\top \begin{bmatrix} A^\top X + X^\top A + C_1^\top Q C_1 & X^\top B_1 + C_1^\top Q D_{11} \\ B_1^\top X + D_{11}^\top Q C_1 & D_{11}^\top Q D_{11} - \gamma^2 P \end{bmatrix} W_V < 0, \tag{3.6}$$

$$A_2^\top X + X^\top A_2 + X^\top R_2 X + Q_2 < 0, \tag{3.7}$$

where

$$V = [C_2, D_{21}], \qquad A_2 = A_1 + B_{11} R_1^{-1} D_{11}^\top Q_1 C_1, \qquad A_1 = A - B_2 R_0^{-1} D_{12}^\top Q C_1,$$

$$R_2 = B_{11} R_1^{-1} B_{11}^\top - B_2 R_0^{-1} B_2^\top, \qquad B_{11} = B_1 - B_2 R_0^{-1} D_{12}^\top Q D_{11},$$

$$Q_2 = C_1^\top (Q_1 + Q_1 D_{11} R_1^{-1} D_{11}^\top Q_1) C_1,$$

is consistent for X and S. Then there exists a static regulator (3.2) for which the closed system (3.3) is admissible and has the performance criterion $J < \gamma$. The matrix of this regulator can be constructed in the form

$$K = K_*(I_l + D_{22}K_*)^{-1}, \qquad \det\big(I_l + D_{22}K_*\big) \neq 0,$$

where K_* is the solution of the matrix inequality (3.4).

The generalized lemma on matrix indeterminacy enables one to construct a family of regulators (3.2) with ellipsoidal set of feedback matrices, i.e., to determine the guaranteed limits of robustness of the required static regulator.

Lemma 3.2 [13]. *Suppose that the following matrix inequality is true:*

$$\Omega = \begin{bmatrix} W & U^\top & V^\top \\ U & R - P_0 & D^\top \\ V & D & -Q_0^{-1} \end{bmatrix} < 0,$$

where $W = W^\top < 0$, $U, V, D, R = R^\top \geq 0$, $P_0 = P_0^\top > 0$, and $Q_0 = Q_0^\top > 0$ are matrices of the corresponding orders. Then, for any matrix K that belongs to the ellipsoid $\mathcal{K}_0 = \big\{K\colon\ K^\top P_0 K \leq Q_0\big\}$, the relations $\rho(KD) < 1$ and

$$W + U^\top \mathbf{D}(K)V + V^\top \mathbf{D}^\top(K)U + V^\top \mathbf{D}^\top(K)\, R\, \mathbf{D}(K)V < 0,$$

where $\mathbf{D}(K) = (I - KD)^{-1}K$, are true.

By using Lemmas 2.4 and 3.2 and condition (3.4) for a closed system, we arrive at the following assertion:

Theorem 3.2. *Suppose that there exist matrices X, $S = S^\top \geq 0$, $P_0 = P_0^\top > 0$, and $Q_0 = Q_0^\top > 0$ satisfying the system of relations (2.5), (2.8), and*

$$\Omega(X, P_0, Q_0) = \begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_3 \\ \Omega_2^\top & \Omega_4 & \Omega_5 \\ \Omega_3^\top & \Omega_5^\top & \Omega_6 \end{bmatrix} < 0,$$

where

$$\Omega_1 = A^\top X + X^\top A + C_1^\top Q C_1 + C_2^\top Q_0 C_2, \qquad \Omega_2 = X^\top B_1 + C_1^\top Q D_{11} + C_2^\top Q_0 D_{21},$$

$$\Omega_3 = X^\top B_2 + C_1^\top Q D_{12} + C_2^\top Q_0 D_{22}, \qquad \Omega_4 = D_{11}^\top Q D_{11} + D_{21}^\top Q_0 D_{21} - \gamma^2 P,$$

$$\Omega_5 = D_{11}^\top Q D_{12} + D_{21}^\top Q_0 D_{22}, \qquad \Omega_6 = D_{12}^\top Q D_{12} + D_{22}^\top Q_0 D_{22} - P_0.$$

Then, for any control (3.2) with $K \in \mathcal{K}_0$, the closed system (3.3) is admissible and has the performance criterion $J < \gamma$.

Let $K = K_1$ be the matrix of regulator (3.2) obtained according to Theorem 3.1. Substituting the expression $u = K_1 y + v$ in Eq. (3.1), we arrive at a similar system with new control vector v. We apply Theorem 3.2 to this system. As a result, for system (3.1), we obtain a family of regulators (3.2) with ellipsoidal set of feedback matrices

$$\mathcal{K}_1 = \{K:\ (K - K_1)^\top P_0(K - K_1) \le Q_0\}$$

for which the closed system is admissible and has the performance criterion $J < \gamma$.

3.2. Dynamic Regulator. Consider system (3.1) with a dynamic regulator

$$\dot{\xi} = Z\xi + Vy, \qquad u = U\xi + Ky, \quad \xi(0) = 0, \tag{3.8}$$

where $\xi \in \mathbb{R}^p$, p is the order of the regulator, and Z, V, U, and K are matrices that should be determined. The combined system (3.1), (3.8) can be rewritten in the form of a similar system in the extended phase space $\mathbb{R}^{n+p}$:

$$\begin{aligned} &\widehat{E}\dot{\widehat{x}} = \widehat{A}\widehat{x} + \widehat{B}_1 w + \widehat{B}_2\widehat{u}, \qquad \widehat{x}(0) = \widehat{x}_0,\\ &z = \widehat{C}_1\widehat{x} + \widehat{D}_{11}w + \widehat{D}_{12}\widehat{u},\\ &\widehat{y} = \widehat{C}_2\widehat{x} + \widehat{D}_{21}w \end{aligned} \tag{3.9}$$

by using a static regulator

$$\widehat{u} = \widehat{K}_*\widehat{y}, \qquad \det(I_m - KD_{22}) \ne 0, \tag{3.10}$$

where

$$\widehat{x} = \begin{bmatrix} x \\ \xi \end{bmatrix}, \qquad \widehat{x}_0 = \begin{bmatrix} x_0 \\ 0 \end{bmatrix}, \qquad \widehat{y} = \begin{bmatrix} y - D_{22}u \\ \xi \end{bmatrix}, \qquad \widehat{u} = \begin{bmatrix} u \\ \dot{\xi} \end{bmatrix}, \qquad \widehat{E} = \begin{bmatrix} E & 0 \\ 0 & I_p \end{bmatrix},$$

$$\widehat{A} = \begin{bmatrix} A & 0_{n\times p} \\ 0_{p\times n} & 0_{p\times p} \end{bmatrix}, \qquad \widehat{B}_1 = \begin{bmatrix} B_1 \\ 0_{p\times s} \end{bmatrix}, \qquad \widehat{B}_2 = \begin{bmatrix} B_2 & 0_{n\times p} \\ 0_{p\times m} & I_p \end{bmatrix},$$

$$\widehat{C}_1 = [C_1, 0_{k\times p}], \qquad \widehat{D}_{11} = D_{11}, \qquad \widehat{D}_{12} = [D_{12}, 0_{k\times p}],$$

$$\widehat{C}_2 = \begin{bmatrix} C_2 & 0_{l\times p} \\ 0_{p\times n} & I_p \end{bmatrix}, \qquad \widehat{D}_{21} = \begin{bmatrix} D_{21} \\ 0_{p\times s} \end{bmatrix}, \qquad \widehat{K}_* = \begin{bmatrix} K_* & U_* \\ V_* & Z_* \end{bmatrix} = (I_{m+p} - \widehat{K}\widehat{D}_{22})^{-1}\widehat{K},$$

$$\widehat{K} = \begin{bmatrix} K & U \\ V & Z \end{bmatrix} = (I_{m+p} + \widehat{K}_*\widehat{D}_{22})^{-1}\widehat{K}_*, \qquad \widehat{D}_{22} = \begin{bmatrix} D_{22} & 0_{l\times p} \\ 0_{p\times m} & 0_{p\times p} \end{bmatrix}. \tag{3.11}$$

In this case, the closed system (3.9), (3.10) takes the form

$$\widehat{E}\dot{\widehat{x}} = \widehat{A}_*\widehat{x} + \widehat{B}_* w, \qquad z = \widehat{C}_*\widehat{x} + \widehat{D}_* w, \qquad \widehat{x}(0) = \widehat{x}_0, \tag{3.12}$$

where

$$\widehat{A}_* = \widehat{A} + \widehat{B}_2 \widehat{K}_* \widehat{C}_2, \qquad \widehat{B}_* = \widehat{B}_1 + \widehat{B}_2 \widehat{K}_* \widehat{D}_{21},$$

$$\widehat{C}_* = \widehat{C}_1 + \widehat{D}_{12} \widehat{K}_* \widehat{C}_2, \qquad \widehat{D}_* = \widehat{D}_{11} + \widehat{D}_{12} \widehat{K}_* \widehat{D}_{21}.$$

Since $\xi_0 = 0$, its performance criterion $\widehat{J}$ of type (2.4) with weight matrix

$$\widehat{X}_0 = \widehat{E}^\top \widehat{H} \widehat{E}, \qquad \widehat{H} = \begin{bmatrix} H & H_1^\top \\ H_1 & H_2 \end{bmatrix} > 0,$$

is independent of H_1 and H_2 and its value coincides with J.

Theorem 3.3. *Suppose that conditions (3.5) are satisfied and the system of relations (2.5), (2.8), (3.6), and*

$$A_2^\top G + G^\top A_2 + G^\top R_2 G + Q_2 < 0, \tag{3.13}$$

$$X - G = \Theta E, \qquad \Theta = \Theta^\top \geq 0, \qquad \operatorname{rank} \Theta \leq p, \tag{3.14}$$

where the matrices A_2, R_2, *and* Q_2 *are given by (3.7), is consistent with respect to* X, G, S, *and* Θ. *Then there exists a dynamic regulator (3.8) for which the closed system (3.12) is admissible and possesses the performance criterion* $J < \gamma$.

Proof. We rewrite conditions (2.5), (2.6), and (2.8) for system (3.12) in the form

$$0 \leq \widehat{E}^\top \widehat{X} = \widehat{X}^\top \widehat{E} \leq \gamma^2 \widehat{X}_0, \qquad \operatorname{rank}\left(\widehat{E}^\top \widehat{X} - \gamma^2 \widehat{X}_0\right) = \rho + p, \tag{3.15}$$

$$\widehat{W} + \widehat{U}^\top \widehat{K}_* \widehat{V} + \widehat{V}^\top \widehat{K}_*^\top \widehat{U} + \widehat{V}^\top \widehat{K}_*^\top \widehat{R} \widehat{K}_* \widehat{V} < 0, \tag{3.16}$$

where

$$\widehat{W} = \begin{bmatrix} \widehat{A}^\top \widehat{X} + \widehat{X}^\top \widehat{A} + \widehat{C}_1^\top Q \widehat{C}_1 & \widehat{X}^\top \widehat{B}_1 + \widehat{C}_1^\top Q \widehat{D}_{11} \\ \widehat{B}_1^\top \widehat{X} + \widehat{D}_{11}^\top Q \widehat{C}_1 & \widehat{D}_{11}^\top Q \widehat{D}_{11} - \gamma^2 P \end{bmatrix}, \qquad \widehat{X} = \begin{bmatrix} X & X_3 \\ X_1 & X_2 \end{bmatrix},$$

$$\widehat{U} = \left[\widehat{B}_2^\top \widehat{X} + \widehat{D}_{12}^\top Q \widehat{C}_1,\, \widehat{D}_{12}^\top Q \widehat{D}_{11}\right], \qquad \widehat{V} = \left[\widehat{C}_2,\, \widehat{D}_{21}\right], \qquad \widehat{R} = \widehat{D}_{12}^\top Q \widehat{D}_{12} \geq 0.$$

It is easy to see that, under these conditions, the matrix $\widehat{X}$ and its diagonal blocks must be nonsingular and, in addition,

$$X_1 = X_3^\top E \qquad \text{and} \qquad X_2 = X_2^\top > 0.$$

We use the conditions (a) and (d) of Lemma 3.1 for the consistency of the matrix inequality (3.16):

$$W_{\widehat{V}}^\top \widehat{W} W_{\widehat{V}} < 0, \qquad W_{\widehat{U}_0}^\top \left(\widehat{W} - \widehat{U}^\top \widehat{R}^+ \widehat{U}\right) W_{\widehat{U}_0} < 0, \qquad \widehat{U}_0 = W_{\widehat{R}}^\top \widehat{U}. \tag{3.17}$$

The first matrix inequality in (3.17) takes the form (3.6) because

$$\widehat{V} = \begin{bmatrix} C_2 & 0_{l\times p} & D_{21} \\ 0_{p\times n} & I_p & 0_{p\times s} \end{bmatrix}, \qquad W_{\widehat{V}} = \begin{bmatrix} I_n & 0_{n\times s} \\ 0_{p\times n} & 0_{p\times s} \\ 0_{s\times n} & I_s \end{bmatrix} W_V, \qquad V = \begin{bmatrix} C_2 & D_{21} \end{bmatrix}.$$

We transform the second matrix inequality in (3.17) by using the relations

$$W_{\widehat{R}} = \begin{bmatrix} 0_{m\times p} \\ I_p \end{bmatrix}, \qquad W_{\widehat{U}_0} = \begin{bmatrix} I_n & 0_{n\times s} \\ -X_2^{-1}X_1 & 0_{p\times s} \\ 0_{s\times n} & I_s \end{bmatrix}, \qquad \widehat{U}_0 = \begin{bmatrix} X_1 & X_2 & 0_{p\times s} \end{bmatrix},$$

$$\widehat{R}^+ = \begin{bmatrix} R_0^{-1} & 0_{k\times p} \\ 0_{p\times k} & 0_{p\times p} \end{bmatrix}, \qquad \widehat{U} W_{\widehat{U}_0} = \begin{bmatrix} B_2^\top G + D_{12}^\top Q C_1 & D_{12}^\top Q D_{11} \\ 0_{p\times n} & 0_{p\times s} \end{bmatrix},$$

$$W_{\widehat{U}_0}^\top \widehat{W} W_{\widehat{U}_0} = \begin{bmatrix} A^\top G + G^\top A + C_1^\top Q_1 C_1 & G^\top B_1 + C_1^\top Q D_{11} \\ B_1^\top G + D_{11}^\top Q C_1 & D_{11}^\top Q D_{11} - \gamma^2 P \end{bmatrix},$$

where

$$G = X - X_3 X_2^{-1} X_1 = X - \Theta E \qquad \text{and} \qquad \Theta = \Theta^\top = X_3 X_2^{-1} X_3^\top \geq 0.$$

As a result, we obtain relation (3.14) and the matrix inequality

$$\begin{bmatrix} A_1^\top G + G^\top A_1 + C_1^\top Q_1 C_1 - G^\top B_2 R_0^{-1} B_2^\top G & G^\top B_{11} + C_1^\top Q_1 D_{11} \\ B_{11}^\top G + D_{11}^\top Q C_1 & -R_1 \end{bmatrix} < 0.$$

By the Schur lemma, this inequality is equivalent to inequality (3.13).

If we find the matrices $X,\ G,\ S,$ and Θ under the indicated conditions of the theorem, then we can determine the complementary blocks $X_1,\ X_2,$ and X_3 of the matrix $\widehat{X}$ by using the spectral decomposition of the matrix $\Theta = \Theta^\top \geq 0$ and the dependence $X_1 = X_3^\top E$. In this case, the rank of the matrix Θ gives the least order of the required dynamic regulator. To satisfy the rank condition in (3.15), we can set

$$H_1 = \gamma^{-2} X_3^\top \quad \text{and} \quad H_2 = \gamma^{-2} X_2 + I_p.$$

The theorem is proved.

Remark 3.1. It is possible to show that, under appropriate conditions, the matrix X in the formulations of Lemma 2.4 and Theorems 3.1–3.3 can be constructed in the form

$$X = S_0 E + E_0 F,$$

where $0 < S_0 = S_0^\top < \gamma^2 H,\ E_0 = W_{E^\top},$ and $F \in \mathbb{R}^{(n-\rho)\times n}$. Furthermore,

$$S = E^\top S_0 E \geq 0 \qquad \text{and} \qquad S - \gamma^2 X_0 = E^\top (S_0 - \gamma^2 H) E \leq 0.$$

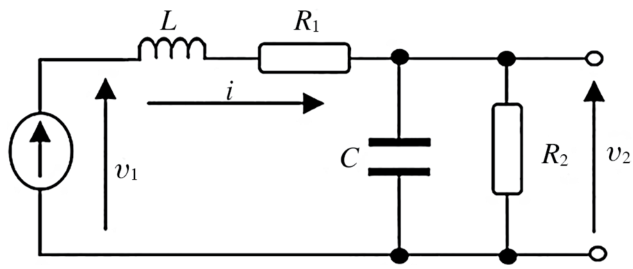

Fig. 1. Electric circuit.

The proof of Theorem 3.3 gives the following algorithm for the construction of the dynamic regulator (3.8):

(1) determination of the matrices X, G, S, and Θ from the system of relations (2.5), (2.8), (3.6), (3.13), and (3.14);

(2) construction of the spectral decomposition of the nonnegative-definite matrix $\Theta = T\Lambda T^\top$, where $T \in \mathbb{R}^{n\times r}$, $\Lambda = \mathrm{diag}\{\theta_1, \ldots, \theta_r\} > 0$, and $r = \mathrm{rank}\,\Theta$;

(3) formation of the complementary blocks $X_1 = T^\top E$, $X_2 = \Lambda^{-1}$, and $X_3 = T$ of the matrix $\widehat{X}$ for $p = r$;

(4) solution of the matrix inequality (3.16) for $\widehat{K}_*$ and determination of the matrix coefficients of regulator (3.8) by using relation (3.11).

4. Example

Consider an electric circuit depicted in Fig. 1 and a control system for this system of the form (3.1) with matrices [27]

$$E = \begin{bmatrix} L & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} -R_1 & -1 & 1 \\ 0 & -1/R_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad D_{11} = D_{21} = D_{22} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

where

$$x = \begin{bmatrix} i & v_2 & v_1 \end{bmatrix}^\top, \qquad z = \begin{bmatrix} v_2 & u + \alpha v_1 \end{bmatrix}^\top, \qquad y = \begin{bmatrix} v_2 & v_1 \end{bmatrix}^\top,$$

$L = 3$ is the inductance, $C = 2$ is the capacitance, $R_1 = 2$ and $R_2 = 1$ are the resistances, i is a current, v_1 and v_2 are voltages, u is a control signal of the current source with bounded disturbances w, and $\alpha = 1$ is a parameter (see Fig. 1). In this system, the pair of matrices (E, A) is impulsive and the triples (E, A, B_2) and (E, A, C_2) are I-controlled and I-observed, respectively.

We choose the following weight matrices of the performance criterion (2.4):

$$P = 1, \quad Q = I_2, \quad \text{and} \quad X_0 = E^\top E.$$

By using the Mathcad Prime software, for $\gamma = 0.7,$ we obtain the matrix

$$X = \begin{bmatrix} 0.53135 & 0.00692 & 0 \\ 0.01038 & 0.97992 & 0 \\ -0.15290 & 0.03987 & -0.88512 \end{bmatrix},$$

satisfying the system of relations (2.5), (2.8), (3.6), and (3.7), and the matrix of the static regulator (3.2)

$$K_1 = -\begin{bmatrix} 0.89374 & 1.00974 \end{bmatrix}$$

for which the closed system (3.3) is admissible and possesses the performance criterion $J = 0.59259 < \gamma.$ In this case, $J_0 = \|G_{zw}\|_\infty = 0.27633.$ The results of numerical experiments demonstrate that a decrease in the parameter α on the segment $[0, 1]$ leads to the elevation of the minimal possible characteristics J_0 and J of the closed system in the case of application of static regulators of type (3.2).

By using Theorem 3.2, we construct an ellipsoidal set of matrices of regulator (3.2) of the form

$$\mathcal{K}_1 = \left\{K : \, (K - K_1)\,\Gamma\,(K - K_1)^\top \le 1\right\}, \qquad \Gamma = \begin{bmatrix} 27.11679 & -0.09483 \\ -0.09483 & 27.36519 \end{bmatrix},$$

for which the closed system (3.3) is admissible and has the performance criterion $J < \gamma.$

Further, for the closed system with regulator matrix $K_1,$ we construct the worst disturbance

$$w = K_0 x, \quad K_0 = \begin{bmatrix} 0.18527 & 0.38263 & 0.00013 \end{bmatrix} \tag{4.1}$$

and the worst initial vector $x_0 = \begin{bmatrix} 0.04910 \; 0.38187 \; -0.23294 \end{bmatrix}^\top$ for the performance criterion J (see Lemma 2.5). The behavior of the solution of the closed system with the worst disturbance

$$E\dot{x} = A_0 x, \qquad x(0) = x_0, \tag{4.2}$$

where

$$A_0 = A + B_2 K_1 C_2 + B_1 K_0,$$

is shown in Fig. 2. The function of this disturbance (4.1) is depicted in Fig. 3. System (4.2) is admissible and has a finite spectrum $\sigma(F_0) = \left\{-0.71783 \pm 0.45121\, i\right\}.$ Its solution has the form

$$x(t) = T\widetilde{x}(t),$$

where T is a matrix of full rank satisfying the relations

$$A_0 T = ET\Lambda \qquad \text{and} \qquad \operatorname{rank} T = \operatorname{rank}(ET) = \operatorname{rank} E$$

and $\widetilde{x}(t)$ is the solution of the ordinary system $\dot{\widetilde{x}} = \Lambda \widetilde{x}.$ Moreover, $\sigma(\Lambda) = \sigma(F_0)$ and $x_0 = T\widetilde{x}_0.$

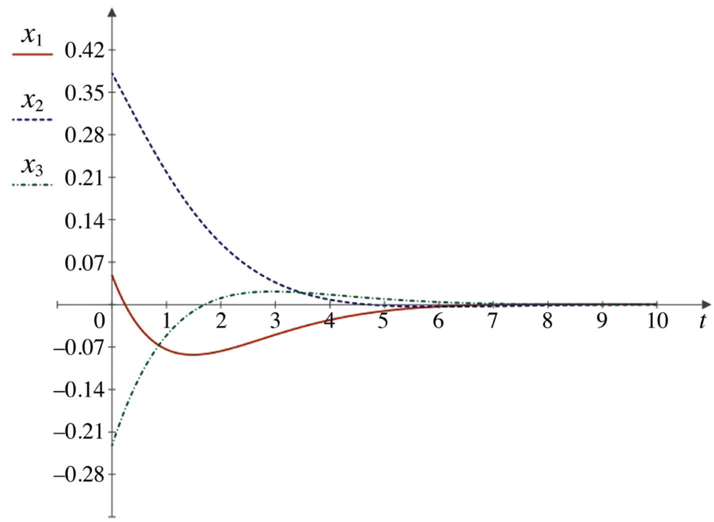

Fig. 2. Behavior of the closed system.

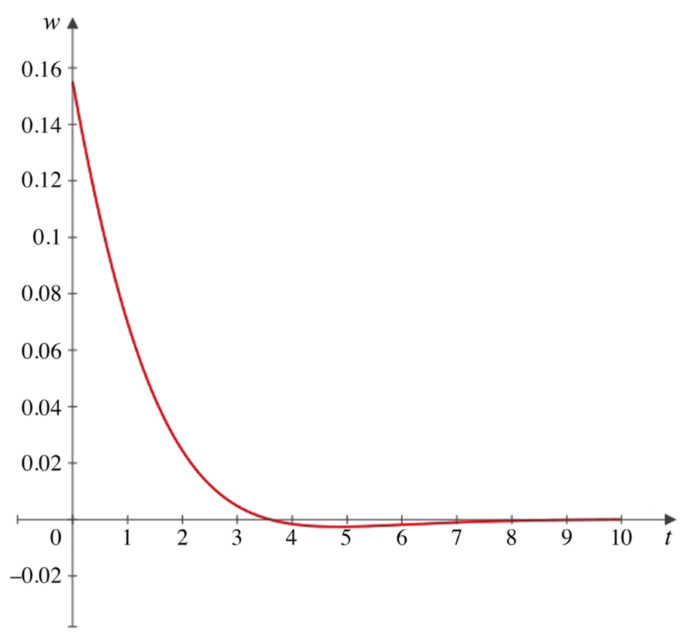

Fig. 3. Worst disturbance for the performance criterion J.

By using Theorem 3.3, we also determine the matrices of the dynamic regulator (3.8)

$$Z = \begin{bmatrix} -0.38338 & -0.00721 \\ 0.08507 & -0.41471 \end{bmatrix}, \qquad V = \begin{bmatrix} -0.03600 & -0.07555 \\ -0.00232 & -0.00068 \end{bmatrix},$$

$$U = \begin{bmatrix} 0.57234 & 2.61512 \end{bmatrix}, \qquad K = \begin{bmatrix} -0.08957 & -0.96093 \end{bmatrix},$$

for which the closed system (3.12) is admissible and has the performance criterion $J = 0.47138$.

REFERENCES

1. S. Campbell, A. Ilchmann, V. Mehrmann, and T. Reis (editors), *Applications of Differential-Algebraic Equations: Examples and Benchmarks*, Springer Nature, Switzerland AG (2019).
2. A. Ilchmann and T. Reis (editors), *Surveys in Differential-Algebraic Equations III*, Springer Internat. Publ., Switzerland (2015).
3. G.-R. Duan, *Analysis and Design of Descriptor Linear Systems*, Springer, New York (2010).
4. R. Riaza, *Differential-Algebraic Systems. Analytical Aspects and Circuit Applications*, World Scientific, Singapore (2008).
5. A. A. Belov and A. P. Kurdyumov, *Descriptor Systems and Control Problems* [in Russian], Fizmatlit, Moscow (2015).
6. A. M. Samoilenko, M. I. Shkil', and V. P. Yakovets', *Linear Systems of Differential Equations with Degenerations* [in Ukrainian], Vyshcha Shkola, Kyiv (2000).
7. F. R. Gantmakher, *Theory of Matrices* [in Russian], Nauka, Moscow (1988).
8. S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishman, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia (1994).
9. G. E. Dullerud and F. G. Paganini, *A Course in Robust Control Theory. A Convex Approach*, Springer, Berlin (2000)
10. B. T. Polyak and P. S. Shcherbakov, *Robust Stability and Control* [in Russian], Nauka, Moscow (2002).
11. D. V. Balandin and M. M. Kogan, *Synthesis of the Regularities of Control Based on Linear Matrix Inequalities* [in Russian], Fizmatlit, Moscow (2007).
12. D. V. Balandin and M. M. Kogan, "Generalized H_∞-optimal control as a compromise between H_∞-optimal and γ-optimal controls," *Avtomat. Telemekh.*, No. 6, 20–38 (2010).
13. A. G. Mazko, *Robust Stability and Stabilization of Dynamical Systems. Methods of Matrix and Conic Inequalities* [in Russian], Institute of Mathematics, National Academy of Sciences of Ukraine, Kyiv (2016).
14. A. G. Mazko and S. N. Kusii, "Stabilization with respect to output and weighted suppression of disturbances in discrete control systems," *Probl. Uprav. Inform.*, No. 6, 78–93 (2017).
15. O. H. Mazko and S. M. Kusii, "Weighted damping of bounded disturbances in the airplane control system in the mode of landing," in: *Proc. of the Institute of Mathematics, National Academy of Sciences of Ukraine* [in Ukrainian], **15**, No. 1 (2018), pp. 88–99.
16. A. G. Mazko and T. O. Kotov, "Robust stabilization and weighted damping of bounded disturbances in descriptor control systems," *Ukr. Mat. Zh.*, **71**, No. 10, 1374–1388 (2019); ***English translation:*** *Ukr. Math. J.*, **71**, No. 10, 1572–1589 (2020).
17. O. H. Mazko and T. O. Kotov, "Estimation of the influence and damping of bounded disturbances in descriptor control systems," in: *Proc. of the Institute of Mathematics, National Academy of Sciences of Ukraine* [in Ukrainian], **16**, No. 2 (2019), pp. 63–84.
18. P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control," *Internat. J. Robust Nonlin. Control*, **4**, 421–448 (1994).
19. S. Xu, J. Lam, and Y. Zou, "New versions of bounded real lemmas for continuous and discrete uncertain systems," *Circuits, Systems, Signal Process*, **26**, 829–838 (2007).
20. M. Chadli, P. Shi, Z. Feng, and J. Lam, "New bounded real lemma formulation and H_∞ control for continuous-time descriptor systems," *Asian J. Control*, **20**, No. 1, 1–7 (2018).
21. F. Gao, W. Q. Liu, V. Sreeram, and K. L. Teo, "Bounded real lemma for descriptor systems and its application," in: *IFAC 14th Triennial World Congress (Beijing, P. R. China)* (1999), pp. 1631–1636.
22. I. Masubushi, Y. Kamitane, A. Ohara, and N. Suda, "H_∞ control for descriptor systems: a matrix inequalities approach," *Automatica*, **33**, No. 4, 669–673 (1997).
23. A. G. Mazko, "Evaluation of the weighted level of damping of bounded disturbances in descriptor systems," *Ukr. Mat. Zh.*, **70**, No. 11, 1541–1552 (2018); ***English translation:*** *Ukr. Math. J.*, **70**, No. 11, 1777–1790 (2019).
24. Yu Feng and M. Yagoubi, *Robust Control of Linear Descriptor Systems*, Springer Nature, Singapore (2017).
25. M. Inoue, T. Wada, M. Ikeda, and E. Uezato, "Robust state-space H_∞ controller design for descriptor systems," *Automatica*, **59**, 164–170 (2015).
26. D. Cobb, "Robust controllability, observability, and duality in singular systems," *IEEE Trans. Automat. Control*, **29**, 1076–1082 (1984).
27. K. Takaba, "Robust H^2 control of descriptor system with time-varying uncertainty," *Internat. J. Robust Nonlin. Control*, **71**, No. 4, 559–579 (1998).