

Stability, Oscillations and Optimization of Systems
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Stability, Oscillations and Optimization of Systems

Volume 2

MATRIX EQUATIONS, SPECTRAL PROBLEMS AND STABILITY OF DYNAMIC SYSTEMS

A.G. Mazko

Matrix equations, Spectral problems and Stability of Dynamic Systems
The volume contains the methods for localization of eigen values of matrices and matrix functions, based on the construction and study of the generalized Lyapunov equation. The theory of linear equations and operators in a matrix space is developed and the known theorems on the inertia of Hermitian solutions of matrix equations are generalized. The author develops new algebraic methods for stability analysis, and an evaluation of spectrum and representation of solutions of linear arbitrary order differential and difference systems.

This monograph is intended for researchers, engineers, and post-graduates interested in the theory of stability and stabilization of dynamic systems, matrix analysis and its applications.

About the Author

A.G.Mazko is Leading researcher, Institute of Mathematics, National Academy of Sciences of Ukraine, Kiev. He is the author or coauthor of more than 75 research papers, including the monograph Localization of Spectrum and Stability of Dynamic Systems (Institute of Mathematics, National Academy of Sciences of Ukraine, 1999). Dr. Mazko received a PhD degree in physics and mathematics from the Institute of Mathematics, National Academy of Sciences of Ukraine, Kiev in 1995.

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Co-Editors

P.Borne

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A.A. Martynyuk and V.G. Miladzhyanov

Matrix Equations, Spectral Problems and Stability of Dynamic Systems

A.G. Mazko

Institute of Mathematics

National Academy of Sciences of Ukraine

Kiev, Ukraine



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Production management by Out of House Publishing Solutions

Printed and bound by TJ International Ltd, Padstow, Cornwall, UK

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British Library Cataloguing in Publication Data

A catalogue record for this book has been requested

Library of Congress Cataloging in Publication Data

A catalogue record has been requested

ISBN 978-1-904868-52-1

Cambridge Scientific Publishers Ltd
PO Box 806
Cottenham, Cambridge CB4 8RT
UK

www.cambridgescientificpublishers.com

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