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On the invariance, integrability and linearization of a generalized modified Emden type equation

During the past three decades or so, considerable attention has been paid to study the dynamics of nonlinear systems. The mathematical model for many important physical systems leads to nonlinear differential equations. Identifying integrable nonlinear dynamical systems and exploring their underlying solutions is one of the challenging problems in nonlinear physics. Different methods have been proposed in order to identify new integrable cases and understand the underlying dynamics associated with the finite dimensional nonlinear dynamical systems. The most widely used methods include Painlevé analysis, direct method, Lie symmetry analysis, Noether's theorem and direct linearization. In this paper, we consider a generalized modified Emden type equation (GMEE),

$$\ddot{x} + (k_1 x + k_2)\dot{x} + \frac{k_1^2}{9}x^3 + \frac{k_1 k_2}{3}x^2 + \lambda_1 x + \lambda_2 = 0,$$

where $k_i's$ and $\lambda_i's$, $i = 1, 2$ are arbitrary constants, which includes several physically important oscillators and systems such as the simple harmonic oscillator, displaced damped harmonic oscillator, Emden type equation and its hierarchy, and so on, and investigate the Lie point symmetries, integrability and linearization. We show that the GMEE admits eight Lie point symmetry generators satisfying $sl(3, R)$ algebra. We carry out the Painlevé singularity structure analysis for a rather general equation. By using extended Prelle-Singer procedure, we derive integrating factors and integrals of motion of this equation and obtain the general solution. We also explore its general solution by using nonlocal transformation. Finally, we show that under certain parametric choice the GMEE exhibits unusual nonlinear dynamical properties.