A fully quantum method of determination of penetrability and reflection coefficients in quantum FRW model with radiation*

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In the paper the closed Friedmann–Robertson–Walker model with quantization in the presence of a positive cosmological constant and radiation is studied [1, 2, 3, 4, 5, 6, 7]. For analysis of the tunneling probability for the birth of an asymptotically deSitter, inflationary Universe as a function of the radiation energy, a new definition of a "free" wave propagating in strong fields is proposed [8]. On such a basis, a new fully quantum approach for determination of incident, reflected and transmitted waves relatively a barrier is constructed, tunneling boundary condition [9, 10] is corrected. A stationary method of determination of penetrability and reflection relatively the barrier with analysis of uniqueness of solution is developed. At the first time non-zero interference between the incident and reflected waves has been taken into account which turns out to play a huge role inside cosmological potentials. For its estimation the coefficient of mixing is introduced. According to the calculations, inside whole region of energy of radiation the tunneling probability for the birth of an asymptotically deSitter, inflationary Universe is very close to its value, obtained in semiclassical approach [11, 12], but essentially differs on the estimations obtained before by known quantum non-stationary approach [13]. The reflection from the barrier in the internal region is determined at first time (which is essentially differs on 1 at the energy of radiation close to the barrier height). Here, modulus of the coefficient of mixing is less 10^{-19} for all energies, that points out that *there is no interference between the found incident and reflected waves and this confirms correctness of proposed definition of "free" wave inside strong fields.* The proposed method is easily generalized on the cosmological models with the barriers of arbitrary shape, what was demonstrated for the FRW-model above with included Chaplygin gas [14] (see also [15, 16]. Results is stable for different variations of studied barriers and the accuracy is 11–18 digits for all coefficients and energies below a barrier height (see tables below).

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Energy	Penetrability $P_{\text{penetrability}}$			Time τ	
	Full QM method 1	Full QM method 2	Method WKB	Full QM method 1	Method WKB
10.0	8.1070×10^{-33}	7.6149×10^{-31}	1.4522×10^{-31}	$8.6156 \times 10^{+32}$	$4.8094 \times 10^{+31}$
20.0	7.6221×10^{-30}	7.7349×10^{-29}	1.4692×10^{-29}	$9.4313 \times 10^{+29}$	$4.8928 \times 10^{+29}$
30.0	7.6975×10^{-29}	7.3089×10^{-27}	1.3848×10^{-27}	$9.5988 \times 10^{+28}$	$5.3354 \times 10^{+27}$
40.0	3.5680×10^{-26}	6.5169×10^{-25}	1.2298×10^{-25}	$2.1257 \times 10^{+26}$	$6.1670 \times 10^{+25}$
50.0	5.5831×10^{-25}	5.4707×10^{-23}	1.0285×10^{-23}	$1.3936 \times 10^{+25}$	$7.5647 \times 10^{+23}$
60.0	2.0591×10^{-22}	4.3423×10^{-21}	8.1523×10^{-22}	$3.8719 \times 10^{+22}$	$9.7797 \times 10^{+21}$
70.0	3.0663×10^{-21}	3.3043×10^{-19}	6.1642×10^{-20}	$2.6640 \times 10^{+21}$	$1.3251 \times 10^{+20}$
80.0	1.5530×10^{-18}	2.3850×10^{-17}	4.4346×10^{-18}	$5.3862 \times 10^{+18}$	$1.8862 \times 10^{+18}$
90.0	1.3181×10^{-17}	1.6564×10^{-15}	3.0658×10^{-16}	$6.4948 \times 10^{+17}$	$2.7923 \times 10^{+16}$
100.0	3.2922×10^{-14}	1.1053×10^{-13}	2.0304×10^{-14}	$2.6622 \times 10^{+14}$	$4.3167 \times 10^{+14}$
110.0	4.9414×10^{-14}	7.0911×10^{-12}	1.2935×10^{-12}	$1.8158 \times 10^{+14}$	$6.9367 \times 10^{+12}$
120.0	8.6052×10^{-11}	4.4005×10^{-10}	7.9523×10^{-11}	$1.0678 \times 10^{+11}$	$1.1555 \times 10^{+11}$
130.0	2.1009×10^{-10}	2.6431×10^{-8}	4.7194×10^{-9}	$4.4822 \times 10^{+10}$	$1.9953 \times 10^{+9}$
140.0	2.2012×10^{-8}	1.5460×10^{-6}	2.7128×10^{-7}	$4.3888 \times 10^{+8}$	$3.5612 \times 10^{+7}$
150.0	2.8361×10^{-6}	8.8293×10^{-5}	1.5114×10^{-5}	$3.4994 \times 10^{+6}$	$6.5663 \times 10^{+5}$
160.0	2.9685×10^{-5}	4.9980×10^{-3}	8.1663×10^{-4}	$3.4471 \times 10^{+5}$	$1.2530 \times 10^{+4}$
170.0	3.4894×10^{-3}	2.6078×10^{-1}	4.2919×10^{-2}	$3.0460 \times 10^{+3}$	$2.4820 \times 10^{+2}$

Table 1: The penetrability $P_{\text{penetrability}}$ of the barrier and duration τ of the formation of the Universe defined in [14] for the FRW-model with the Chaplygin gas obtained in the fully quantum and semiclassical approaches (minimum of the hole is -93.579 and its coordinate is 1.6262, maximum of the barrier is 177.99 and its coordinate is 5.6866): the fully QM method 1 is calculations by the fully quantum approach for the boundary located in the coordinate of the minimum of the internal hole (i. e. coordinate is 1.6262), the fully QM method 2 is calculations by the fully quantum approach for the boundary located in the internal turning point $a_{\text{tp, in}}$

Energy	Fully quantum method				Turning points	
	Penetrability	Reflection	Interference	Summation	$a_{\text{tp, in}}$	$a_{\text{tp, out}}$
10.0	$8.1070216824 \times 10^{-33}$	1.00000000000000	3.06×10^{-20}	1.00000000000000	3.49	7.08
20.0	$7.6221543404 \times 10^{-30}$	1.00000000000000	8.55×10^{-20}	1.00000000000000	3.59	7.05
30.0	$7.6975296835 \times 10^{-29}$	1.00000000000000	1.82×10^{-20}	1.00000000000000	3.69	7.01
40.0	$3.5680158760 \times 10^{-26}$	1.00000000000000	2.34×10^{-19}	1.00000000000000	3.79	6.97
50.0	$5.5831154210 \times 10^{-25}$	1.00000000000000	4.98×10^{-20}	1.00000000000000	3.89	6.92
60.0	$2.0591415452 \times 10^{-22}$	1.00000000000000	4.86×10^{-20}	1.00000000000000	3.98	6.88
70.0	$3.0663252971 \times 10^{-21}$	1.00000000000000	1.84×10^{-19}	1.00000000000000	4.08	6.83
80.0	$1.5530040238 \times 10^{-18}$	1.00000000000000	2.08×10^{-19}	1.00000000000000	4.18	6.78
90.0	$1.3181086626 \times 10^{-17}$	1.00000000000000	5.03×10^{-20}	1.00000000000000	4.28	6.73
100.0	$3.2922846164 \times 10^{-14}$	0.99999999999996	3.13×10^{-20}	1.00000000000000	4.38	6.67
110.0	$4.9414392175 \times 10^{-14}$	0.99999999999995	1.45×10^{-19}	1.00000000000000	4.48	6.61
120.0	$8.6052092530 \times 10^{-11}$	0.99999999991394	1.06×10^{-19}	1.00000000000000	4.59	6.55
130.0	$2.1009662247 \times 10^{-10}$	0.99999999978990	2.68×10^{-20}	1.00000000000000	4.70	6.48
140.0	$2.2012645564 \times 10^{-8}$	0.99999997798735	1.60×10^{-19}	1.00000000000000	4.83	6.39
150.0	$2.8361866579 \times 10^{-6}$	0.99999716381330	4.89×10^{-20}	0.99999999999996	4.96	6.30
160.0	$2.9685643504 \times 10^{-5}$	0.99997031435611	6.94×10^{-20}	0.99999999999961	5.11	6.18
170.0	$3.4894544195 \times 10^{-3}$	0.99651054553176	2.02×10^{-19}	0.99999999995131	5.31	6.02

Table 2: The coefficients of the penetrability, reflection and mixing calculated by the fully quantum method and test on their summation for the FRW-model with the Chaplygin gas density component (the fully quantum approach 1 is used at the internal boundary located in the coordinate of the minimum of the internal hole)