CAPTURE OF DARK MATTER BY THE SOLAR SYSTEM

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1. Introduction

Dark matter density in our Galaxy is

$$\rho_g \simeq 0.4 \cdot 10^{-24}\ \mathrm{g/cm^3}\,. \tag{1}$$

Only upper limits on the level of 10^{-19} $\mathrm{g/cm^3}$ are known for dm density ρ_{SS} in Solar System. Information on ρ_{SS} is important for experiments aimed at detection of dm.

According to Xu, Siegel, dm density, captured from Galaxy, at the Earth orbit is $\sim 10^{-20}$ $\mathrm{g/cm^3}$, within only an order of magnitude from best upper limits on it!

2. Total mass of captured dark matter

SS is immersed in halo of dark matter (dm) and moves together with it around center of Galaxy. In reference frame, comoving with halo, velocities v of dm particles in halo have Maxwell distribution

$$f(v)\,dv = \sqrt{54/\pi}\,(v^2 dv/u^3)\exp\left(-3v^2/2u^2\right) \longrightarrow \sqrt{54/\pi}\,(v^2 dv/u^3);$$

local rms velocity $u \simeq 220$ km/s is large as compared to typical planetary $v \simeq 30$ km/s.

Particle cannot be captured by Sun alone. Interaction with planet is necessary for it, this is three-body problem. Capture is dominated by particles whose orbits are close to parabolic ones with respect to the Sun since their trajectories are most sensitive to additional attraction by planet.

Capture is effectively described by restricted three-body problem:

Interaction between heavy bodies (the Sun and a planet) is treated exactly.
As exactly is treated motion of the third, light body (dmp) in gravitational field of two heavy ones.
One neglects back reaction of light particle upon motion of two heavy bodies. This approximation is fully legitimate for our purpose.
Still, restricted three-body problem is rather complicated, and requires here both subtle analytical treatment and serious numerical calculations.

We resort to dimensional estimate for mass of captured dark matter.

Total mass captured by the Sun (its mass is M) together with a planet with mass m_p, during lifetime $T \simeq 4.5 \cdot 10^9$ years $\simeq 10^{17}$ s of SS, is

$$\Delta m_p = \rho_g T < \sigma v > ;$$

σ is capture cross-section. Product σv is averaged here over distribution $f(v)\, dv = \sqrt{54/\pi}\,(v^2 dv/u^3)$.
We estimate $< \sigma v >$ with dimensional arguments, supplemented by two physical requirements: masses m_p and M of two heavy components of restricted three-body problem should enter result symmetrically, and mass of dmp should not enter result at all.

Then final estimate for captured mass is

$$\Delta m_p \sim \rho_g T \sqrt{54\pi}\;\; k^2\, m_p\, M/u^3\,. \tag{2}$$

here k is the Newton gravitation constant; an extra power of π, inserted into this expression, is perhaps inherent in σ.
Since capture would be impossible if the planet were not bound to the Sun, it is only natural that result is proportional to corresponding effective "coupling constant" $k m_p M$.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
present work	$0.22 \cdot \mathbf{10^{18}}$	$3.2 \cdot \mathbf{10^{18}}$	$3.9 \cdot \mathbf{10^{18}}$	$0.42 \cdot \mathbf{10^{18}}$	$1239 \cdot \mathbf{10^{18}}$	$372 \cdot \mathbf{10^{18}}$	$57 \cdot \mathbf{10^{18}}$	$67 \cdot \mathbf{10^{18}}$
Xu, Siegel	$0.42 \cdot \mathbf{10^{20}}$	$3.5 \cdot \mathbf{10^{20}}$	$3.8 \cdot \mathbf{10^{20}}$	$1.2 \cdot \mathbf{10^{20}}$	$49 \cdot \mathbf{10^{20}}$	$28 \cdot \mathbf{10^{20}}$	$12 \cdot \mathbf{10^{20}}$	$16 \cdot \mathbf{10^{20}}$

Table 1

Disagreement is huge for all planets, and especially for light ones where it exceeds two orders of magnitude. We cannot spot exactly its origin since calculations by Xu, Siegel involve numerical simulations. However, we cannot see any reasonable possibility for serious increase of our results.

Moreover, our results can be considered as upper limits for amount of captured dm, at least because we have neglected inverse process, that of captured dmp ejection due to same three-body gravitational interaction.

Total mass of captured dm constitutes according to Table 1 about $1.5 \cdot \mathbf{10^{21}}$ g. It is small as compared to total mass $\sim \mathbf{10^{33}}$ g of common matter in SS. It is small even as compared to total mass of dm in SS: this total mass constitutes $\sim \mathbf{10^{31}}$ g (for effective radius of SS $\sim \mathbf{10^{5}}$ au).
However, it is an order of magnitude larger than dm mass inside radius of Neptune orbit $\boldsymbol{r_N \approx 30}$ au.
Contribution to discussed effect of diffuse (non-dark) matter in SS is much smaller since in homogeneous dust gravitational forces acting on dmp are compensated.

3. Distribution of dark matter density

While total masses Δm_p of captured dm can be (hopefully) described by above simple dimensional estimate, situation for corresponding dm densities $\Delta \rho_p$ is more subtle.

Dmp's captured into elliptic trajectories had initially hyperbolic trajectories, focussed at the Sun and close to parabolic ones. Eccentricity e changes from $e = 1 + \varepsilon_1$ to $e = 1 - \varepsilon_2$. $\varepsilon = \varepsilon_1 + \varepsilon_2 \ll 1$ We demonstrate that this captured density is certainly much less:

$$\rho \ll 10^{-21}\ \mathrm{g/cm^3}\,. \tag{3}$$

results from gravitational perturbation by planet, and is proportional to m_p. Thus, ε, $\varepsilon_{1,2} \sim m_p/M$. In capture process impact parameter ρ goes over smoothly into minimum distance to the Sun $r_{\min}$ of elliptic trajectory, $r_{\min} \approx \rho$.

Radius-vector $\boldsymbol{r}$ of captured dmp is related to azimuthal angle ϕ as follows:

$$\boldsymbol{r = p\,/(1 + e\cos\phi)\,,} \tag{4}$$

where $\boldsymbol{p}$ is the so-called orbit parameter. Ratio of maximum and minimum distances is

$$\boldsymbol{r_{\max}/r_{\min} = (1+e)/(1-e)\ \simeq 2/(1-e)\ \simeq M/m_p\,,} \tag{5}$$

or

$$\boldsymbol{r_{\max}\ \simeq r_p(M/m_p)\,,} \tag{6}$$

since we are interested in $\boldsymbol{\rho\ \sim r_p}$.

After summation over all trajectories, captured dmp's fill in sphere of radius $R_p \sim r_p\,(M/m_p)$, and correspondingly, of the volume

$$V_p \sim \frac{4\pi}{3} R_p^3 \sim \frac{4\pi}{3}\, r_p^3\,(M/m_p)^3\,.$$

For the largest planet Jupiter we obtain $a_J \sim 10^3\, r_J$, in good agreement with result of numerical simulations by Petrosky. Thus obtained dm density in SS is small, much less than Galactic one.

Let us make at last the assumption resulting in the most optimistic prediction for the "partial" dark matter densities $\Delta\rho_p$.

We assume that each of total masses $\boldsymbol{\Delta m_p}$ of captured dm occupies volume $\boldsymbol{(4\pi/3)r_p^3}$. We do not claim that this assumption is correct, but believe, however, that comparison of its (almost certainly, overoptimistic) results with observational limits will be instructive. Corresponding values of "partial" dark matter densities $\boldsymbol{\Delta\rho_p = \Delta m_p/(4\pi r_p^2/3)}$ (in g/cm^3) are presented in Table 2.

We omit in it densities due to Uranus and Neptune, tiny even on discussed scale. In accordance with accepted model, total dark matter density $\boldsymbol{\rho}_{\rm dm}$ at given radius does not coincide with corresponding $\boldsymbol{\Delta\rho_p}$. It includes, in line with it, sum of contributions to density due to all planets, outer with respect to given one.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn
$\boldsymbol{\Delta\rho_p}$	$2.7\cdot\mathbf{10^{-22}}$	$6.0\cdot\mathbf{10^{-22}}$	$2.7\cdot\mathbf{10^{-22}}$	$8.4\cdot\mathbf{10^{-24}}$	$6.2\cdot\mathbf{10^{-22}}$	$3.0\cdot\mathbf{10^{-23}}$
$\boldsymbol{\rho}_{\rm dm}$	$1.8\cdot\mathbf{10^{-21}}$	$1.5\cdot\mathbf{10^{-21}}$	$9.3\cdot\mathbf{10^{-22}}$	$6.6\cdot\mathbf{10^{-22}}$	$6.5\cdot\mathbf{10^{-22}}$	$3.0\cdot\mathbf{10^{-23}}$

Table 2

4. Observational upper limits on the density of dark matter

Most reliable and accurate information on dark matter in our Solar System follows from studies of perihelion precession of Venus, Earth, and Mars.
Under assumptions that dm density ρ_{dm} is distributed spherically symmetric with respect to the Sun and that eccentricity of planetary orbit is small, relative shift of perihelion per period is (Khriplovich):

$$\frac{\delta\phi}{2\pi} = -\frac{2\pi\rho_{\mathrm{dm}}(r)r^3}{M},$$

where r is orbit radius.

Recent, most precise observational data (Pitjeva) on precession of perihelia are presented in Table 3 (therein theoretical values $\delta\phi_{\rm th}$ of perihelion rotation and results of observations $\delta\phi_{\rm obs}$ are given in angular seconds per century).

	Venus	Earth	Mars
$\delta\phi_{\rm th}$	8.6248	3.8388	1.3510
$\delta\phi_{\rm obs}$	8.6247 ± 0.0002	3.8390 ± 0.0003	1.3512 ± 0.0003

Table 3

With these data, one arrives at upper limits on dm density at distances from the Sun, corresponding to orbit radii of Venus, Earth, and Mars, on the level of

$$\boldsymbol{\rho}_{\rm dm} < \mathbf{2 \cdot 10^{-19}}\ \mathrm{g/cm^3}\,\mathbf{.}$$

This observational upper limit exceeds by about two orders of magnitude results (almost certainly overestimated) presented in Table 2.

5. Summary

Our results do not mean, however, that the searches for the dark matter in the Solar System are senseless. Of course, the capture of the Galactic dark matter analyzed here is not the only conceivable source of the dark matter in the Solar System. It is quite possible in particular that the Solar System itself has arisen due to a local high-density fluctuation of the dark matter.