Oksana Ye. Hentosh
Dept. of Nonlin. Math. Analysis,
IAPMM, NAS of Ukraine,
3B Naukova str., Lviv, 79060, Ukraine
E-mail: ohen@ua.fm

THE LAX INTEGRABLE DIFFERENTIAL-DIFFERENCE DYNAMICAL SYSTEMS ON EXTENDED PHASE SPACES

The Lax type flows in the forms [1, 2]

$$L_{t_p} = [(L^p)_+, L], \quad p \in \mathbb{N}, \tag{1}$$

on the dual space $\mathcal{G}^*$ to the Lie algebra of linear operators

$$L := \sum_{j<\infty} u_j(n) T^j,$$

where $u_j \in C^\infty(\mathbb{Z}/q\mathbb{Z}; \mathbb{R})$, $j \in \mathbb{Z}$, $q \in \mathbb{N}$, and T is the shift operator, satisfying the following rule

$$T^j u = (T^j u) T^j,$$

and the lower index $+$ signs a projection of the corresponding operator on the Lie subalgebra $\mathcal{G}_+ \subset \mathcal{G}$, which consists of the elements $\sum_{0 \le j < \infty} u_j(n) T^j$, with respect to the scalar product

$$(A, B) := \sum_{n \in \mathbb{Z}/q\mathbb{Z}} \sum_{j \in \mathbb{Z}} a_j(n) b_j(n), \quad A, B \in \mathcal{G},$$

$A := \sum_{j<\infty} a_j(n) T^j$, $B := \sum_{i<\infty} T^{-i} b_i(n)$, are considered. The corresponding evolutions for eigenfunctions $f_k \in W := L_\infty(\mathbb{Z}/q\mathbb{Z}; \mathbb{R})$ and adjoint eigenfunctions $f_k^* \in W$, $k = \overline{1, N}$, of the associated with (1) isospectral problem take the forms

$$f_{k,t_p} = ((L^p)_+ f_k), \quad f^*_{k,t_p} = -((L^p)^*_+ f^*_k), \tag{2}$$

where functions $f_k, f_k^* \in W$ are related to $N \in \mathbb{N}$ different eigenvalues.

The existence of Hamiltonian representation for the hierarchy of dynamical systems (1)-(2) on an extended phase space $\mathcal{G}^* \times W^{2N}$ is investigated by use of the invariant Casimir functionals' property under the Lie-Backlund transformation on the space $\mathcal{G}^*$

$$L_{>0} \mapsto L = L_{>0} + \sum_{k=1}^{N} f_k T (T-1)^{-1} f_k^*,$$

where $L_{>0} := \sum_{0<j<\infty} u_j(n) T^j$. The corresponding hierarchies of Lax type additional symmetries [3] are stated to be Hamiltonian too. It is established that the additional symmetry is generated by the Poisson structure, being equal to the tensor product of the R-deformed canonical Lie-Poisson bracket [1] on $\mathcal{G}^*$ and the standard Poisson bracket on the space W^{2N}, and some power of a suitable spectral eigenvalue is its Hamiltonian function.

The similar problems for the central extension [3, 4] of the Lie algebra $\mathcal{G}$ are studied also.

References

[1] Blaszak M. and Marciniak K. R-matrix approach to lattice integrable systems. *J. Math. Phys.*, 1994. – **35**, No. 9. – P. 4661-4682.

[2] Hentosh O., Prytula M., Prykarpatsky A. *Differential- Geometric and Lie-Algebraic Foundations of Studying Integrable Nonlinear Dynamical Systems on Functional Manifolds*, Lviv National University Publishing, Lviv, 2006, 406 pp. (in Ukrainian)

[3] Hentosh O.Ye. and Prykarpatsky A.K. Integrable three-dimensional coupled nonlinear dynamical systems related with centrally extended operator Lie algebras. *Opuscula Mathematica*, 2007. – **27**, No. 2. – P. 231-244.

[4] Blaszak M. and Szum A. Lie algebraic approach to the construction of $(2+1)$-dimensional lattice-field and field integrable Hamiltonian equations. *J. Math. Phys.*, 2001. – **42**, No. 1. – P. 225-259.